Engineering 3 (2017) 888–891

Contents lists available at ScienceDirect

Engineering

journal homepage: www.elsevier.com/locate/eng

Research
Tunnel Engineering—Article

Universal Method for the Prediction of Abrasive Waterjet Performance in Mining

Eugene Averin

OOO Skuratovsky Experimental Plant, Tula 300911, Russia

ARTICLE INFO

Article history:
Received 22 March 2017
Revised 8 September 2017
Accepted 5 December 2017
Available online 8 December 2017

Keywords:
Abrasive waterjet
Energy conservation approach
Depth of cut
Fracture mechanics
Threshold velocity
Mining

ABSTRACT

Abrasive waterjets (AWJs) can be used in extreme mining conditions for hard rock destruction, due to their ability to effectively cut difficult-to-machine materials with an absence of dust formation. They can also be used for explosion, intrinsic, and fire safety. Every destructible material can be considered as either ductile or brittle in terms of its fracture mechanics. Thus, there is a need for a method to predict the efficiency of cutting with AWJs that is highly accurate irrespective of material. This problem can be solved using the energy conservation approach, which states the proportionality between the material removal volume and the kinetic energy of AWJs. This paper describes a method based on this approach, along with recommendations on reaching the most effective level of destruction. Recommendations are provided regarding rational ranges of values for the relation of abrasive flow rate to water flow rate, standoff distance, and size of abrasive particles. I also provide a parameter to establish the threshold conditions for a material's destruction initiation based on the temporary-structural approach of fracture mechanics.

1. Introduction

The need to increase the efficiency of mining machines and extend the field of their application has come to a head. Special attention is also required in the creation and development of mining equipment that provides an increase in technical and economic performance along with secure labor conditions. Waterjet technologies are one of the most promising solutions to meet these needs. These technologies are based on the usage of the energy of high-speed water streams to create stress within a destructed material that is higher than the strength of that material, leading to the propagation of cracks and to erosion [1,2]. Waterjet technologies are currently in broad usage in highly technological industries [3–5] due to their advantages, which include high machining versatility, the ability to contour, no thermal distortion, and a small cutting force. They also provide the possibility to cut difficult-to-machine materials such as ceramics, marbles, and layered composites. In mining, waterjet technologies allow an increase in productivity by permitting advancement to occur several times faster, and permit the destruction of hard rocks without dust formation but with explosion, intrinsic, and fire safety [6,7]. For these reasons, waterjet technologies can be exceptionally useful in extreme mining conditions and for the destruction of hard rocks.

The effectiveness of these technologies increases sharply with the addition of an abrasive to the water stream [6,8–11]. For rock destruction, the most common usage of abrasive waterjets (AWJs) is in the processing of natural stones, especially marbles and granite [12,13]. Possible ways of applying AWJs in underground mining have also been considered [6,14–16]. For the purpose of increasing the efficiency and safety of the working processes in mines, it is reasonable to use AWJs in the following operations:

- Dismantling works (i.e., cutting metal structures, armored cable, steel cord conveyor belts, etc.);
- Contouring the face preparatory workings when installing fasteners;
- Repairing excavations and restoring the area of their cross-section;
- Weakening hard rock with discharge slots, with further destruction by mechanical tools;
- Cutting rock and solid materials, including high-strength metals in extreme conditions (in areas of geological faults, fractured rock mass, etc.); and
- Drilling gas drainage holes in order to prevent outbursts in coal mines.

As seen from this list of mining conditions, it is necessary to effectively cut not only rock, but also metals, concrete, and other solid materials with different physical and mechanical properties.

E-mail address: evgeniy.averin.90@mail.ru

https://doi.org/10.1016/j.eng.2017.12.004
2095-8099/

2. Basic principles of abrasive waterjets

There are two ways in which abrasives are mixed with water to form AWJs: the direct pumping system and the entrainment system. In a direct pumping system, abrasives are pre-mixed with water to form slurry, which is then pumped and expelled through a nozzle. In an entrainment system, a high-pressure waterjet is first formed by an orifice; next, abrasives are entrained into the waterjet to mix with it and form AWJs. For mining conditions, entrainment systems are more convenient because of their relative cheapness and compactness, and their lower requirement for a specific quantity of metal to be cut [17].

To be more precise, AWJs in entrainment systems are formed as follows: A high-pressure pump compresses water to a high (140–420 MPa) or ultra-high (over 420 MPa) pressure. A water supply system then delivers water to a cutting head, where a waterjet forms. This waterjet goes to a mixing chamber. Abrasive is also transported to the mixing chamber from a container using a special delivery system. In the mixing chamber, the waterjet mixes with abrasive particles to form slurry. The slurry then goes to a convergent section of the mixing chamber and further to a collimator, where the final forming of the AWJ occurs.

In most of the calculations associated with this technology, it is possible to neglect the influence of the water, since its main function is to accelerate the particles within the formed AWJs and then carry them off the surface of the destructed material. The destruction of materials with AWJs is caused mainly by the impact of abrasive particles in the stream [6]. Thus, this process can be described as an interaction between the abrasive particles and the material's surface. This type of interaction depends on the type of destructed material—that is, on whether the material is ductile or brittle. This division is due to a modern conception about the toughness of materials, which is based on fracture mechanics. In this conception, destruction is considered to be a form of erosion, which consists of simultaneous deformations, such as elastic and bound deformations and cracking.

It is now possible to define the type of material under certain loading and environmental conditions at the atomic level, using the temporary-structural approach of fracture mechanics [18]. Brittle fracture is related to the rupture of chemical bonds, whereas ductile fracture is due to the shift of atomic planes along slip planes. For this reason, most materials with isotropic features, which are characterized by a random arrangement of atoms, demonstrate brittle fracture. In contrast, solids with a crystalline structure, in which jointing is difficult because of the elimination of local stresses by elastic deformations, mostly demonstrate ductile fracture. Thus, hard rocks under normal conditions are always brittle, whereas most metals are ductile.

3. Method to predict abrasive waterjet performance

Although the destruction of hard rock is considered to be the main application for AWJ in mining, this technology can also be used (as mentioned above) to cut non-rock materials. Therefore, there is a need for a method to predict the efficiency of cutting with AWJs that is highly accurate irrespective of material. This problem can be solved using energy conservation modeling. This approach is based on the assumption that there is proportionality between the material removal volume and the kinetic energy of AWJ. This approach has led to the development of several semi-empirical methods that allow the depth of cut to be determined for various materials.

The first of these methods was developed by Blickwedel et al. [19], and takes into account the exponential character of traverse speed and its influence on the depth of the cut, d:

$$d = C_s \frac{P - P_c}{u^f} \tag{1}$$

where C_s is the empirical parameter, which depends on the material's properties; f is the empirical parameter, which describes the energy losses of the jet within a cut slit; u is the traverse speed of the AWJ; P_c is the minimum water pressure required to initiate destruction; and P is the current water pressure.

Parameter f is determined by the following equation:

$$f = 0.86 + \frac{2.09}{u} \tag{2}$$

This approach was developed further by Chen et al. [20] for cutting aluminum ceramic, which is a brittle material. The main feature of this method is the introduction of the abrasive flow rate, m_a:

$$d = 0.0101 \frac{m_a P}{u^{0.78}} \tag{3}$$

Subsequent development of this approach was done by extending the number of parameters included; the approach was then applied to various materials. For example, Wang [21] added the jet diameter, d_j, and the water density, ρ_w, to this method and used it to predict the cutting efficiency of polymer matrix composites, which are ductile materials.

$$d = 0.6267 \frac{m_a^{0.407} P}{d_j u^{0.637} \rho_w} \tag{4}$$

Although the exponents for traverse speed in Eqs. (3) and (4) are distinct from those in Eq. (2), the former are more reliable and can be considered as constants, for a certain type of destructed material [6]. Thus, for brittle materials, the exponent is approximately equal to 0.78, whereas for ductile materials, the exponent is about 0.64.

The constant values before the fractions in both equations are the coefficients of machinability for a certain material. The following formula, which comes from Ref. [4], can be used for the primary assessment of this coefficient:

$$C_s = 3.626 \times 10^{-8} \exp(-2.448 \times 10^{-8} \sigma_c) \tag{5}$$

where σ_c is the uniaxial compressive strength.

Although Eq. (5) is quite useful, it is preferable to detect the machinability factor for a particular material of interest using the regression method, which requires experimental studies.

It is known that effective cutting angles depend on the material's type [22–24]. The most effective destruction of a brittle material occurs when the cutting angle, φ, is 90°, whereas for ductile materials, the optimum cutting angle is about 20°. In order to take into consideration the influence of the angle of a waterjet attack on cutting efficiency, a new coefficient, k_φ, is enacted, which can have a value between 0 and 1. Such a coefficient was determined [22] by approximating functions built on generalized experimental data [6,22–24].

For brittle materials:

$$k_\varphi = 0.99 \exp\left(-0.5 \left|\frac{\varphi - 90}{28.4}\right|^{1.77}\right) \tag{6}$$

For ductile materials:

$$k_\varphi = 8437 \sin\left(\frac{\varphi}{68049}\right) \exp\left(\frac{-\varphi}{20.5}\right) \tag{7}$$

Eq. (6) operates for values from 0° to 180°, and Eq. (7) operates for values from 0° to 90°.

The next point that should be discussed is the abrasive flow rate. Studies on this parameter [25–28] have shown that its increase leads to an increase in cutting efficiency, until it reaches a surplus level, at which point the cutting efficiency decreases. It was established that the optimum flow rate does not relate to the material. In fact, the optimum flow rate depends on the momentum transfer efficiency of the waterjet energy to the abrasive particles; that is, the optimum flow rate is a relation of the abrasive flow rate, m_a, to the water flow rate, m_w [29]. This optimality means that under the given conditions, the cutting process provides the deepest depth of cut with the least possible specific energy [6]. As shown in Ref. [25], the optimum flow rate lies in the range of $m_a/m_w = 0.1–0.35$. The function of the cutting efficiency, which depends on the relation of the abrasive flow rate to the water flow rate, is quite a flat curve (i.e., it does not increase sharply to the turning point from both sides of the function), and has no wide scatter within the range given above. Thus, keeping the abrasive flow rate on such a level that the relation m_a/m_w is within the described range allows the achievement of at least 90% maximal cutting efficiency. Taking into account the fact that the kinetic energy of the waterjet is effectively transferred to the abrasive particles, the AWJ speed just after leaving the tool is half of the waterjet speed at the immediate point of entry of the tool [25]. Thus, the theoretical maximum efficiency of the transfer of kinetic energy from the waterjet to the abrasive particles is 25%. If the relation m_a/m_w is not maintained within the established range, the actual kinetic energy of the abrasives is less than 10% of the total energy of the waterjet in an AWJ system [30].

The waterjet speed, u_w, can be calculated by the following equation:

$$u_{\mathrm{w}} = \mu\sqrt{\frac{2P}{\rho_{\mathrm{w}}}} \tag{8}$$

where μ is a coefficient for the velocity loss due to friction between the water flow and the orifice wall.

The abovementioned developments have led to the following mode of this method; this mode is based on the energy conservation approach and can be recommended for practical usage:

$$d = \frac{\mu^2 m_{\mathrm{a}} P}{4\rho_{\mathrm{w}} d_{\mathrm{j}} u^f} C_{\mathrm{s}} k_{\varphi} \tag{9}$$

4. Results and discussion

Eq. (9) does not take into account significant parameters such as type of abrasive, size of abrasive particles, and standoff distance [31]. However, these parameters can be "introduced" into the method by means of recommendations. In mining conditions, it is impossible to reuse abrasive, so the cost of abrasive should be kept as low as possible. The most reasonable material for abrasive is quartz sand. This recommendation is confirmed by the study described in Ref. [32], in which 52 types of abrasive were examined with a variety of parameters (cost, effectiveness, grain toughness, etc.). Unlike garnet, chrysolite, or metal shot, when used as an abrasive, quartz sand does not provide high-quality contouring of a worked material; nevertheless, it is still quite effective for destruction. Regarding the size of abrasive particles, reasonable values are from 0.1 mm to 0.25 mm [6,33].

In order to provide the most efficient destruction, a standoff distance of 4–6 mm is recommended [6]. At smaller distances, the spatter of water with abrasive and destructed material particles impedes effective cutting by reaching the tool's output orifice and causing its soiling. At greater distances, dissipation of the AWJ occurs, along with a decrease in its velocity due to the interaction of abrasive particles with the environment (i.e., energy loss caused by friction with the mine air).

Note that Eq. (1) contains a parameter for the minimum water pressure that is required to initiate destruction. Although subsequent modes of this approach, including Eq. (9), do not include the threshold parameters, it is clear that in order to initiate destruction, certain special conditions are needed. It is known [34] that dynamic fracture (and rock destruction by AWJs is definitely a dynamic process) cannot be predicted on the basis of classical fracture mechanics. Numerous experimental results [35,36] reveal contradictions with conventional approaches (i.e., critical stress or critical stress intensity factor concepts) that can only be explained by the inapplicability of static approaches to dynamic problems [37]. Fracture on each scale level is a result of complicated kinetic processes such as growth, propagation, and jointing of microcracks [35,36]; therefore, it should never be considered as an instantaneous event. It is possible to take these microscale processes into consideration, and hence to solve the nonlinear problem of dynamic fracture while remaining within the framework of linear formulation by using the fracture criterion based on the incubation time, τ. This criterion was originally proposed in order to predict crack initiation under dynamic conditions, as formulated in Ref. [38]. It can be successfully used to predict fracture initiation in brittle solids [39,40]. The incubation time characterizes the duration of preparation of the medium to the fracture or phase transition, and is a material constant [41]. A relatively simple method of measuring the incubation time involves considering it as the time required for relaxation of the tensile stress at a point distant from the fracture interface [37].

This criterion makes it possible to obtain the threshold velocity of a particle that is needed to initiate the growth of cracks in the destructed material. For a ductile material [41], the threshold velocity, V_c, can be estimated from the following equation:

$$V_{\mathrm{c}} = 335.5\left(\frac{R}{\tau}\right)^5 \frac{\pi\rho(1-\nu^2)}{E} \tag{10}$$

where R is the medium radius of the abrasive particle, ρ is the particle density, ν is the Poisson ratio of the destructed material, and E is Young's modulus of the destructed material.

For brittle materials [42], the threshold velocity can be estimated from the following equation:

$$V_{\mathrm{c}} = 0.63\sqrt[9]{\frac{J_{\mathrm{c}}^5 E^2}{\sigma_{\mathrm{c}}^5 \tau^5 (1-\nu^2)^2 \rho^2}} \tag{11}$$

where J_c is the critical value of the J integral [43,44].

By doubling the estimated value of the threshold velocity obtained from Eq. (10) or Eq. (11)—depending on the type of material—in order to calculate the waterjet velocity (if the condition regarding the optimal relation of the abrasive flow rate to the water flow rate is completed), it is possible to obtain the critical value of the water pressure that is required to initiate destruction using Eq. (8).

5. Conclusions

This paper describes a method based on the energy conservation approach for predicting the depth of the cut when cutting either ductile or brittle materials with AWJs. A comparison of experimental data with theoretical values obtained using the represented method is provided in Table 1. For calculation purposes, the water density, ρ_w, was taken as 1000 kg m^{-3}; the coefficient for the velocity loss due to friction between the water flow and the orifice wall, μ, was 0.75; the cutting angle for brittle materials was 90°; and the cutting angle for ductile materials was 20°.

Although this method includes several restrictions regarding the relation of the abrasive flow rate to the water flow rate, the standoff distance, the size of abrasive particles, and the type of abrasive, these restrictions lead to the most effective destruction.

Table 1
Comparison of experimental and theoretical values of depth of cut for various materials.

Material (type)	σ_c (MPa)	P (MPa)	d_j (mm)	U (mm·s^{-1})	m_a (kg·s^{-1})	d_{exp} (mm)	d_{th} (mm)	R^2
Marble (brittle)	27.2	50–150	3.5–7.5	0.7–7.0	5–30	6–410	6–405	0.93
Limestone (brittle)	90.1	50–150	3.5–7.5	0.7–7.0	5–30	2–85[a]	2–86	0.98
Granite (brittle)	115.5	50–150	3.5–7.5	0.7–7.0	5–30	1.2–52[a]	1.4–50	0.94
Aluminum (ductile)	150.0	175	0.6	0.3–1.7	48–96	40–200	48–210	0.78

d_{exp}: experimental depth of cut; d_{th}: theoretical depth of cut.
[a] If high traverse speed and low water pressure occurred simultaneously, no destruction occurred.

Thus, they should be considered as recommendations that do not limit the usage of the method but rather provide auxiliary information to increase the productivity of AWJs. This paper also discusses a parameter to establish the threshold conditions for the initiation of a material's destruction.

Acknowledgements

I wish to offer profound thanks to my advisor, Prof. Alexander Zhabin, for having given me the opportunity to work on stimulating investigations at needed times, for guiding and supporting me, for teaching me what is relevant within the research arena and, more importantly, for his patience and his rightful leniency throughout my doctoral studies.

References

[1] Huang L, Folkes J, Kinnell P, Shipway PH. Mechanisms of damage initiation in a titanium alloy subjected to water droplet impact during ultra-high pressure plain waterjet erosion. J Mater Process Technol 2012;212(9):1906–15.
[2] Kong MC, Axinte D, Voice W. Aspects of material removal mechanism in plain waterjet milling on gamma titanium aluminide. J Mater Process Technol 2010;210(3):573–84.
[3] Folkes J. Waterjet—an innovative tool for manufacturing. J Mater Process Technol 2009;209(20):6181–9.
[4] Wang J. Abrasive waterjet machining of engineering materials. Zurich: Trans Tech Publications, Ltd.; 2003.
[5] Albert M. The promise of waterjet technology for micromachining. Mod Mach Shop 2013;85(10):76–81.
[6] Brenner VA, Zhabin AB, Pushkarev AE, Shhegolevskii MM. Abrasive waterjet cutting of rocks. Moscow: Mining Book; 2003. Russian.
[7] Merzlyakov VG, Baftalovsky VE, Baydinov VN. On using hydraulic technology for mechanization of mining operations. Min Equip Electromech 2010;6:2–6. Russian.
[8] Merzlyakov VG, Baftalovsky VE. Development and creation of abrasive waterjet tools for cutting hard materials and estimation of rational parameters of the process. Min Equip Electromech 2008;4:27–31. Russian.
[9] Engin IC. A correlation for predicting the abrasive water jet cutting depth for natural stones. S Afr J Sci 2012;108(9–10). Art.#692.
[10] Hashish M, Steele DE, Bothell DH. Machining with super-pressure (690 MPa) waterjets. Int J Mach Tool Manuf 1997;37(4):465–79.
[11] Susuzlu T, Hoogstrate AM, Karpuschewski B. Initial research on the ultra-high pressure waterjet up to 700 MPa. J Mater Process Technol 2004;149(1–3):30–6.
[12] Aydin G, Karakurt I, Aydiner K. Prediction of the cut depth of granitic rocks machined by abrasive waterjet (AWJ). Rock Mech Rock Eng 2013;46(5):1223–35.
[13] Karakurt I, Aydin G, Aydiner K. An experimental study on the depth of cut of granite in abrasive waterjet cutting. Mater Manuf Process 2012;27(5):538–44.
[14] Sitarama Chakravarthy P, Ramesh Babu N. A new approach for selection of optimal process parameters in abrasive water jet cutting. Mater Manuf Process 1999;14(4):581–600.
[15] Kim JG, Song JJ, Han SS, Lee CI. Slotting of concrete and rock using an abrasive suspension waterjet system. KSCE J Civ Eng 2012;16(4):571–8.
[16] Lu Y, Tang J, Ge Z, Xia B, Liu Y. Hard rock drilling technique with abrasive water jet assistance. Int J Rock Mech Min Sci 2013;60:47–56.
[17] Hashish M. The waterjet as a tool. Proceedings of the 14th international conference on jetting technology; 1998 Sep 21-23; Brugge, Belgium. London: Professional Engineering Publishing; 1998. p. 1–14.
[18] Evstifeev AD, Gruzdkov AA, Petrov YV. Dependence of the type of fracture on temperature and strain rate. Tech Phys 2013;58(7):989–93.
[19] Blickwedel H, Guo NS, Haferkamp H, Louis H. Prediction of abrasive jet cutting performance and quality. In: Saunders D, editor. Jet cutting technology: proceedings of the 10th international symposium; 1990 Oct 31-Nov 2; Amsterdam, the Netherlands. London: Elsevier Applied Science; 1990. p. 163–79.
[20] Chen L, Siores E, Wong WCK. Kerf characteristics in abrasive waterjet cutting of ceramic materials. Int J Mach Tool Manuf 1996;36(11):1201–6.
[21] Wang J. Abrasive waterjet machining of polymer matrix composites—cutting performance, erosive process and predictive models. Int J Adv Manuf Technol 1999;15(10):757–68.
[22] Ives LK, Ruff AW. Transmission and scanning electron microscopy studies of deformation at erosion impact sites. Wear 1978;46(1):149–62.
[23] Hockey BJ, Wiederhorn SM, Johnson H. Erosion of brittle materials by solid particle impact. In: Bradt RC, Hasselman DPH, Lange FF, editors. Fracture mechanics of ceramics, volume 3: flaws and testing. New York: Plenum Press; 1978. p. 379–402.
[24] Sheldon GL, Finnie I. On the ductile behavior of nominally brittle materials during erosive cutting. J Eng Ind 1966;88(4):387–92.
[25] Zhabin AB, Averin EA. Improvement of the method for the cut depth calculation when cutting by hydroabrasive instrument. Min Equip Electromech 2014;11:24–9. Russian.
[26] Yazici S. Abrasive jet cutting and drilling of rock [dissertation]. Rolla: University of Missouri; 1989.
[27] Faber K, Oweinah H. Influence of process parameters on blasting performance with the abrasive jet. In: Saunders D, editor. Jet cutting technology: proceedings of the 10th international symposium; 1990 Oct 31-Nov 2; Amsterdam, the Netherlands. London: Elsevier Applied Science; 1990. p. 365–82.
[28] Chalmers EJ. Effect of parameter selection on abrasive waterjet performance. In: Labus TJ, editor. Proceedings of the 6th American water jet conference; 1991 Aug 24-27; Houston, TX, USA. St. Louis: Water Jet Technology Association; 1991. p. 345–54.
[29] Laurinat A, Louis H, Meier-Wiechert GA. Model for milling with abrasive water jets. In: Hashish M, editor. Proceedings of the 7th American water jet conference; 1993 Aug 28-31; Seattle, WA, USA. St. Louis: Water Jet Technology Association; 1993. p. 119–39.
[30] Hashish M. Pressure effects in abrasive-waterjet (AWJ) machining. J Eng Mater Technol 1989;111(3):221–8.
[31] Zhabin AB, Averin EA. Systematization of parameter of rocks erosion destruction with abrasive waterjet. Min Equip Electromech 2015;4:41–4. Russian.
[32] Hessling M. Recent examination relating to the effects of the abrasive material, operating parameters and rock properties on the depth of cut obtainable with abrasive high presser water jets when cutting rock. In: Wood PA, editor. Symposium proceedings: 9th international symposium on jet cutting technology; 1988 Oct 4–6; Sendai, Japan. Cranfield: BHR Group; 1988. p. 357–76.
[33] Hashish M. Aspects of abrasive-waterjet performance optimization. In: Proceedings of the 8th international symposium on jet cutting technology; 1986 Sep 9–11; Durham, UK. p. 297–308.
[34] Homma H, Shockey DA, Murayama Y. Response of cracks in structural materials to short pulse loads. J Mech Phys Solids 1983;31(3):261–79.
[35] Ravi-Chandar K, Knauss WG. An experimental investigation into dynamic fracture: I. Crack initiation and arrest. Int J Fract 1984;25(4):247–62.
[36] Ravi-Chandar K, Knauss WG. An experimental investigation into dynamic fracture: III. On steady-state crack propagation and crack branching. Int J Fract 1984;26(2):141–54.
[37] Kazarinov NA, Bratov VA, Petrov YV, Fedorovsky GD. Evaluation of fracture incubation time from quasistatic tensile strength experiment. Mater Phys Mech 2014;19:16–24.
[38] Petrov YV, Morozov NF. On the modeling of fracture of brittle solids. J Appl Mech 1994;61(3):710–2.
[39] Petrov YV. On the incubation stage of fracture and structural transformations in continuous media under pulse energy injection. Mech Solids 2007;42(5):692–9.
[40] Bratov V. Incubation time fracture criterion for FEM simulations. Acta Mech Sin 2011;27(4):541–9.
[41] Gorbushin NA, Volkov GA, Petrov YV. On the effect of the geometrical shape of a particle on threshold energy in erosion damage. Tech Phys 2013;58(3):388–92.
[42] Zhabin AB, Averin EA. Elementary method for calculation of hard rocks erosion caused by abrasive waterjets. Min Equip Electromech 2015;5:44–8. Russian.
[43] Cherepanov GP. Crack propagation in continuous media. J Appl Math Mech 1967;31(3):503–12.
[44] Rice JR. A path independent integral and the approximate analysis of strain concentration by notches and cracks. J Appl Mech 1968;35(2):379–86.