A new method to calculate the productivity index for vertical fractured well of tight gas reservoir

Li Junjian[1], Jiang Hanqiao[1], Gao Huimei[2], He Yingfu[2]
(1. Faculty of Petroleum Engineering, China University of Petroleum, Beijing 102249, China;
2. Exploration and Development Research Institute of Daqing Oilfield Company, Ltd., Daqing, Heilongjiang 163712, China)

Abstract: Generally the irreducible water saturation of low permeability gas reservoir is quite high which leads to the permeability stress sensibility and threshold pressure gradient. Under the assumption that permeability varies with experimental law of the pseudo pressure drop, according to concepts of perturbable ellipses and equivalent developing regulations, the calculation method of stable production of hydraulically fractured gas well in low permeability reservoirs is investigated with threshold pressure. And productivity curve is drawn and analyzed. The result shows that, permeability modulus and threshold pressure have effect on production of fractured gas well. The higher the permeability modulus and the threshold pressure, the lower the production is. Therefore, the impact of stress sensitive and threshold pressure must be considered when analyzing the productivity of vertical fracture well in low permeability gas reservoir.

Key words: tight gas reservoir; stress sensitive; threshold pressure; vertical fracture well; productivity analysis

1 Introduction

Generally the irreducible water saturation of low permeability gas reservoir is quite high which leads to the permeability stress sensibility and threshold pressure gradient. Most of the tight gas reservoirs are fractured before production[1]. Therefore the investigation of productivity index of vertically fractured well is meaningful. There are a lot of studies on productivity index of vertically fractured well. Jiang Xueting[2] and Wang Yongli[3] adopted conformal transformation to calculate the productivity index of vertically fractured well for oil and gas reservoir. But they did not take permeability stress sensibility and threshold pressure gradient into account. Li Sheng[4] and Yin Hongjun[5] respectively made use of elliptic flow model and bilinear flow model to study the productivity index with threshold pressure gradient. Yang Zhengming[6] wrote that the flow type of certain part of gas is radial flow, whereas other part is linear flow. He built the computation method based on his model. But the model did not consider stress sensibility and the threshold pressure gradient. Beyond those, many scholars also researched the productivity index of vertically fractured well[7-11]. But all of those studies failed to simultaneously take the stress sensibility and the threshold pressure gradient into consideration. Therefore, a new model of productivity index of vertical fractured well with stress sensibility and threshold pressure gradient is established based on perturbable ellipse and the equivalent developmental rectangular.

2 Derivation of the productivity model

When vertically fractured well is working, plane 2-D elliptic seepage will be induced. That is to say, conjugated isobaric ellipse family and hyperbola family whose focuses are the fracture endpoints have been formed. The relationship between rectangular coordinates and elliptical coordinates is shown in Fig. 1.

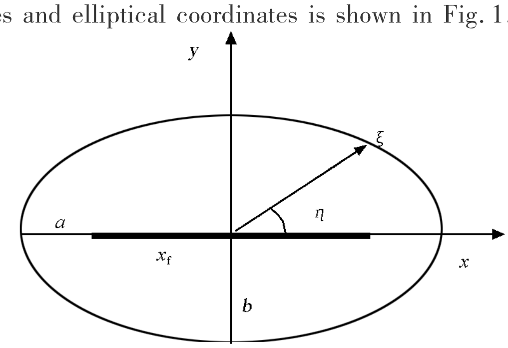

Fig. 1 Relationship between rectangular coordinates and elliptical coordinates

$$x = a\cos\eta \qquad y = b\sin\eta \tag{1}$$

$$a = x_f \operatorname{ch}\xi \qquad b = x_f \operatorname{sh}\xi \tag{2}$$

Where, a and b are long axis and short axis of ellipse individually; x_f is the half length of fracture; ch and sh

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are hyperbolic cosine and hyperbolic sine individually. Besides:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (3)$$

$$\frac{x^2}{(x_f\cos\eta)^2}-\frac{y^2}{(x_f\sin\eta)^2}=1 \quad (4)$$

We could describe isobaric ellipse family by using the developmental rectangle family based on the concept of perturbable ellipse.

$$\bar{x}=x_f\text{ch}\,\xi \quad \bar{y}=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}b\sin\eta\,d\eta=\frac{2x_f}{\pi}\text{sh}\,\xi \quad (5)$$

For the low permeability gas reservoir, generalized Darcy formula could be expressed as

$$v=\frac{K}{\mu_g}\left(\frac{\partial p}{\partial\bar{y}}-\lambda\right) \quad (6)$$

From Darcy law, we know mass flow velocity is

$$\rho v=\rho\frac{K}{\mu_g}\left(\frac{\partial p}{\partial\bar{y}}-\lambda\right)=\frac{\rho_{sc}Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (7)$$

Where, K is zone permeability, mD; μ_g is gas viscosity, mPa· s; λ is threshold pressure gradient, MPa/m; h is effective thickness of strata, m; Q is gas-well production, m^3/d; ρ is gas density, kg/m^3; subscript sc is standard condition. Besides,

$$\frac{dp}{d\bar{y}}=\frac{dp}{d\xi}\frac{d\xi}{d\bar{y}}=\frac{dp}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi} \quad (8)$$

Therefore,

$$\rho\frac{K}{\mu_g}\left(\frac{dp}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi}-\lambda\right)=\frac{\rho_{sc}Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (9)$$

Where,

$$\rho=\frac{pM}{RTZ};\ \rho_{sc}=\frac{p_{sc}M}{RT_{sc}Z_{sc}} \quad (10)$$

Where, M is the molecular weight of gas; R = 8 314 $m^2/(s^2\cdot K)$ is the universal constant of gas; T is the formation temperature; K and Z are the coefficients of gas compressibility; p is the reservoir pressure.

Therefore,

$$\frac{pM}{RTZ}\frac{K}{\mu_g}\left(\frac{dp}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi}-\lambda\right)=\frac{p_{sc}M}{RT_{sc}Z_{sc}}\frac{Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (11)$$

$$K\frac{p}{\mu_gZ}\left(\frac{dp}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi}-\lambda\right)=\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (12)$$

$$K\left(\frac{p}{\mu_gZ}\frac{dp}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi}-\lambda\frac{p}{\mu_gZ}\right)=\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (13)$$

Pseudo-pressure function and pseudo-threshold pressure gradient are defined as

$$m=\int_{p_m}^{p}\frac{p}{\mu_gZ}dp \quad \lambda_m=\frac{p}{\mu_gZ}\lambda \quad (14)$$

$$K\left(\frac{dm}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi}-\lambda_m\right)=\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (15)$$

The stress sensitivity of permeability is considered, and the permeability should satisfy following formula.

$$K=K_ie^{\alpha(m-m_i)} \quad (16)$$

Where, K_i is the initial permeability of stratum, mD; a is permeability modulus, mPa· s/MPa^2; m_i is the initial pseudo-pressure function.

Therefore,

$$K_ie^{\alpha(m-m_i)}\left(\frac{dm}{d\xi}\frac{\pi}{2x_f}\frac{1}{\text{ch}\,\xi}-\lambda_m\right)=\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{4x_fh\text{ch}\,\xi} \quad (17)$$

$$e^{\alpha(m-m_i)}\left(\frac{dm}{d\xi}-\lambda_m\frac{2x_f\text{ch}\,\xi}{\pi}\right)=\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{2\pi K_ih} \quad (18)$$

It will be easier to read if we write

$$A=\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{2\pi K_ih} \qquad \lambda_m\frac{2x_f}{\pi}=B \quad (19)$$

Therefore, Eq. (18) becomes

$$e^{\alpha(m-m_i)}\left(\frac{dm}{d\xi}-B\text{ch}\,\xi\right)=A \quad (20)$$

Moreover, we define function $H=e^{\alpha(m-m_i)}$, therefore,

$$\frac{dH}{d\xi}-B\alpha\text{ch}\,\xi H=\alpha A \quad (21)$$

Obviously, Eq. (21) belongs to the following type.

$$\frac{dy}{dx}+a_1(x)y=h(x) \quad (22)$$

And the solution of Eq. (22) is

$$y(x)=\int_{x_0}^{x}\frac{p(\xi)}{p(x)}h(\xi)d\xi+y_0\frac{p(x_0)}{p(x)} \quad (23)$$

Where,

$$p=e^{\int a_1(x)dx} \quad (24)$$

Therefore, the solution of Eq. (21) is

$$H=\int_{\xi_w}^{\xi}\frac{e^{-\alpha B\text{sh}\,x}}{e^{-\alpha B\text{sh}\,\xi}}\alpha A dx+H_{wf}\frac{e^{-\alpha B\text{sh}\,\xi_w}}{e^{-\alpha B\text{sh}\,\xi}} \quad (25)$$

$$He^{-\alpha B\text{sh}\,\xi}=\alpha A\int_{\xi_w}^{\xi}e^{-\alpha B\text{sh}\,x}dx+H_{wf}e^{-\alpha B\text{sh}\,\xi_w} \quad (26)$$

Which

$$H_{wf}=e^{\alpha(m_{wf}-m_i)}=e^{\alpha(\int_{p_{wf}}^{p}\frac{p}{\mu_gZ}dp-\int_{p_i}^{p}\frac{p}{\mu_gZ}dp)}$$

Where, p_{wf} is the bottom hole pressure and p_i is the initial reservoir pressure.

Substituting A and B into the Eq. (26), obtain

$$He^{-2\alpha\lambda_m\frac{x_f}{\pi}\text{sh}\,\xi}=\alpha\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{2\pi K_ih}\int_{\xi_w}^{\xi}e^{-2\alpha\lambda_m\frac{x_f}{\pi}\text{sh}\,x}dx+H_{wf}e^{-2\alpha\lambda_m\frac{x_f}{\pi}\text{sh}\,\xi_w} \quad (27)$$

Therefore, production formula is obtained.

$$H_ee^{-2\alpha\lambda_m\frac{x_f}{\pi}\text{sh}\,\xi_e}=\alpha\frac{p_{sc}T}{T_{sc}Z_{sc}}\frac{Q_{sc}}{2\pi K_ih}\int_{\xi_w}^{\xi_e}e^{-2\alpha\lambda_m\frac{x_f}{\pi}\text{sh}\,x}dx+H_{wf}e^{-2\alpha\lambda_m\frac{x_f}{\pi}\text{sh}\,\xi_w} \quad (28)$$

$$Q_{sc} = \left[H_e e^{-2\alpha\lambda_m \frac{x_f}{\pi}\operatorname{sh}\xi_e} - H_{wf} e^{-2\alpha\lambda_m\frac{x_f}{\pi}\operatorname{sh}\xi_w}\right]\frac{T_{sc}Z_{sc}}{p_{sc}T}\frac{2\pi K_i h}{\alpha}\frac{1}{\int_{\xi_w}^{\xi_e} e^{-2\alpha\lambda_m\frac{x_f}{\pi}\operatorname{sh}x}\,dx} \quad (29)$$

Where, H_e is the pressure function at external boundary; H_{wf} is the pressure function of sand face pressure.

Considering unit conversion, the final production formula is

$$Q_{sc} = \frac{2\pi K_i h}{\alpha}\frac{T_{sc}Z_{sc}}{p_{sc}T}\frac{H_e e^{-2\alpha\lambda_m \frac{x_f}{\pi}\operatorname{sh}\xi_e} - H_{wf} e^{-2\alpha\lambda_m\frac{x_f}{\pi}\operatorname{sh}\xi_w}}{\int_{\xi_w}^{\xi_e} e^{-2\alpha\lambda_m\frac{x_f}{\pi}\operatorname{sh}x}\,dx} \quad (30)$$

Eq. (30) is the production formula of vertically fractured well of low permeability gas reservoir.

Generally, for elliptic flow we have $\xi_w = 0$. And ξ_e is solved as the following. From the relationship between rectangular coordinates and elliptical coordinates, we could obtain

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (31)$$

Substituting a and b into Eq. (31) individually,

$$\frac{x^2}{(x_f\operatorname{ch}\xi)^2} + \frac{y^2}{(x_f\operatorname{sh}\xi)^2} = 1 \quad (32)$$

ξ_e satisfies Eq. (32) certainly, therefore,

$$\frac{x^2}{(x_f\operatorname{ch}\xi_e)^2} + \frac{y^2}{(x_f\operatorname{sh}\xi_e)^2} = 1 \quad (33)$$

When ξ_e is big,

$$\operatorname{ch}\xi_e = \frac{e^{\xi_e} + e^{\xi_e}}{2} \approx \frac{1}{2}e^{\xi_e} \quad (34)$$

$$\operatorname{sh}\xi_e = \frac{e^{\xi_e} - e^{\xi_e}}{2} \approx \frac{1}{2}e^{\xi_e} \quad (35)$$

Obviously,

$$\frac{x^2}{\left(x_f\frac{1}{2}e^{\xi_e}\right)^2} + \frac{y^2}{\left(x_f\frac{1}{2}e^{\xi_e}\right)^2} = 1 \quad (36)$$

$$x^2 + y^2 = x_f^2\frac{1}{4}e^{2x'_0} \quad (37)$$

$$\xi_e = \ln\frac{2r_e}{x_f} \quad (38)$$

3 Model verification

For a certain vertical fractured gas well, the half length of the facture is 106.7 m, the effective permeability is $0.75 \times 10^{-3}\ \mu m^2$, the reservoir temperature is 364.87 K, the well spacing is 1 200 m, the initial pressure is 27.41 MPa, the net pay is 7.7 m, absolute open flow potential of well testing is $11.8 \times 10^4\ m^3/d$ and the calculated absolute open flow potential is $11.12 \times 10^4\ m^3/d$. Fig. 2 shows good correspondence of different productivity index.

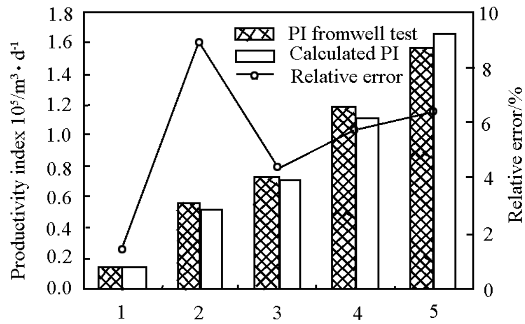

Fig. 2 Contrast between well test absolute open flow potential and calculated productivity index (PI)

4 Affecting factors analysis

Three series of productivity curves of vertical fracture gas well by using the new model are plotted in Fig. 3 to Fig. 5. The net thickness of the reservoir is 10.0 m; the initial effective permeability is $0.65 \times 10^{-3}\ \mu m^2$. The temperature of the reservoir is 379 K; the supply radius is 500 m. The half lengths of the fracture of Fig. 2 and Fig. 3 are 150 m. The permeability multipliers exponent α for Fig. 3 and Fig. 5 are respectively 0.5×10^{-4} mPa • s/MPa^2 and 0.6×10^{-4} mPa • s/MPa^2. The threshold pressure gradient of Fig. 4 and Fig. 5 are 10 MPa^2/(mPa• s).

Some conclusions can be learned from Fig. 3 to Fig. 5, threshold pressure gradient, permeability modulus as well as fractured length affect the productivity index. As the quasi threshold pressure gradient and the permeability modulus become larger, the productivity index becomes lower. The productivity index grows as the length of the fracture grows. So, it is a must to consider the threshold pressure gradient as well as dielectric strain when proceeding in production proration.

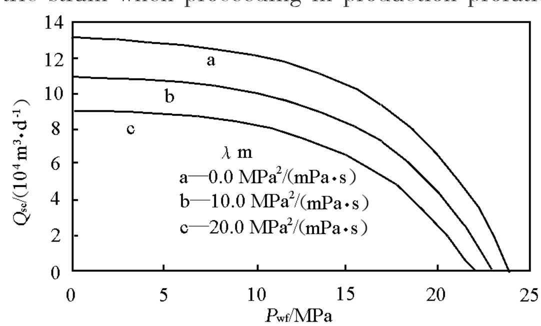

Fig. 3 Impact on productivity index of quasi threshold pressure gradient

5 Conclusions

1) Under the assumption that permeability varies with experimental law of the pseudo pressure drop, ac-

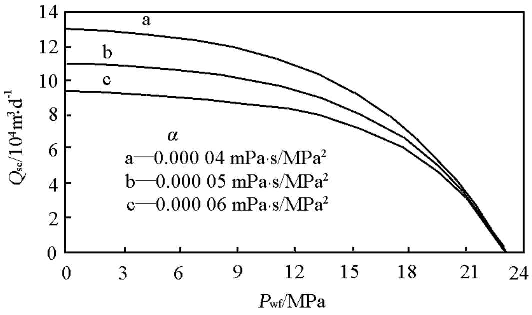

Fig. 4 Impact on productivity index of permeability modulus

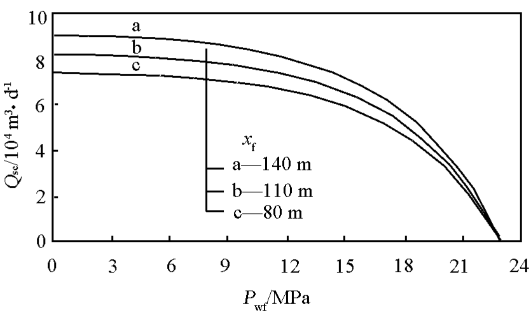

Fig. 5 Impact on productivity index of fracture half length

cording to concepts of perturbable ellipses and equivalent developing regulations, the calculation method of stable production of hydraulically fractured gas well in low permeability reservoirs is investigated with threshold pressure.

2) Threshold pressure gradient, permeability modulus as well as fractured length affect the productivity index.

3) As the quasi threshold pressure gradient and the permeability modulus become larger, the productivity index becomes lower.

4) The productivity index grows as the length of the fracture grows.

5) The impact of stress sensitive and threshold pressure must be considered when analyzing the productivity of vertical fracture well in low permeability gas reservoir.

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Author

Li Junjian, male, was born in 1983 and received his M. S. degree from University of Petroleum, China in 2005. He is studying for his doctoral degree in China University of Petroleum. His research interest is oil/gas field development. He can be reached by E-mail: junjian@126.com