The establishment of a new deliverability equation considering threshold pressure gradient

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Abstract: The flowing mechanism of a low permeability gas reservoir is different from a conventional gas reservoir, especially for that with higher irreducible water saturation the threshold pressure gradient exists. At present, in all the deliverability equation, the additional pressure drop caused by the threshold pressure gradient is viewed as constant, but this method has big error in the practical application. Based on the non-Darcy steady flow equation, the limited integral of the additional pressure drop is solved in this paper and it is realized that the additional pressure drop is not a constant but has something to do with production data, and a new deliverability equation is derived, with the relevant processing method for modified isochronal test data. The new deliverability equation turns out to be practical through onsite application.

Key words: low permeability gas reservoir; threshold pressure gradient; deliverability equation

1 Introduction

The low permeability gas reservoir with high water saturation has the threshold pressure gradient. Because of the threshold pressure gradient, the bottom hole pressure (BHP) of the gas well usually drops fast and gets build-up slowly when shut-in, which causes the deliverability curve abnormal. The present deliverability equation considering the threshold pressure gradient is obtained by adding a constant into the conventional equation, and it has great errors in application. This paper derives a new equation considering the threshold pressure gradient from the right start point of derivation for a flow equation, and proposes a relevant approach of well test analysis, which makes sense for the deliverability evaluation of a low permeability gas reservoir.

2 Error analysis of present deliverability equation

The present deliverability equation considering the threshold pressure gradient is obtained by adding a constant to the conventional equation[1]:

$$p_e^2 - p_{wf}^2 = Aq_g + Bq_g^2 + C$$

Where, p_e is boundary pressure, MPa; p_{wf} is bottom hole flowing pressure, MPa.

$$C = 2\bar{p}\lambda(r_e - r_w)$$

C is a constant independent of production. λ is threshold pressure gradient, MPa/m; r_e is delivery radius, m; r_w is well radius, m.

Take modified isochronal well test as an example to analyze the errors in present equations.

Assuming the test production rate q_i ($i = 1, 2, 3, 4$) satisfies with the following relationship,

$$q_1 < q_2 < q_3 < q_4$$

The 4 BHP then have relationship as following,

$$p_{wf1} > p_{wf2} > p_{wf3} > p_{wf4}$$

Then it is easy to get

$\bar{p}_1 > \bar{p}_2 > \bar{p}_3 > \bar{p}_4$ and $C_1 > C_2 > C_3 > C_4$

If considering C as a constant $\overline{C}$, and it satisfies with $C_4 < \overline{C} < C_1$,

$$(p_e^2 - p_{wf1}^2 - \overline{C})/q_{sc1} > (p_e^2 - p_{wf1}^2 - C_1)/q_{sc1}$$

$$(p_e^2 - p_{wf4}^2 - \overline{C})/q_{sc4} < (p_e^2 - p_{wf4}^2 - C_4)/q_{sc4}$$

Where, q_{sc} is production in standard state, m^3/d.

Reflect them on the deliverability curve (Fig. 1).

Apparently, considering C as a constant could decrease the slope of deliverability curve, and even make it negative, which increases the error of the well test analysis.

3 Establishment of new deliverability equation

The motion equation considering threshold pressure gradient is[4]:

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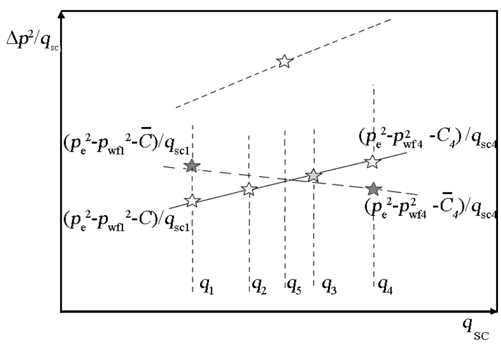

Fig. 1 Analysis on abnormal $\Delta p^2/q_{sc}, q_{sc}$ deliverability curve

$$\begin{cases} v = 0 & (\text{grad}p \leqslant \lambda) \\ v = \dfrac{k}{\mu}(\text{grad}p - \lambda) & (\text{grad}p > \lambda) \end{cases}$$

Assuming a constant pressure outer boundary with a well in the center, by introducing the threshold pressure gradient λ in, the equation with the non-Darcy turbulent flow term is obtained:

$$\frac{dp}{dr} - \lambda = \frac{\mu}{k}v + \beta\rho v^2 \qquad (1)$$

Where, k is permeability, 10^{-3} μm²; β is turbulence coefficient, m^{-1}; μ is gas viscosity in standard state, mPa·s.

Converting the velocity to that under standard surface condition yields:

$$v = \frac{q_r}{2\pi rh} = \frac{q_{sc}}{2\pi rh}\frac{p_{sc}}{Z_{sc}T_{sc}}\frac{ZT}{p}$$

$$\rho = \frac{pMr_g}{ZRT} \qquad (2)$$

Where, h is formation thickness, m; Z_{sc} is the natural gas deviation factor in standard state, zero dimension; T is formation temperature; T_{sc} is standard state temperature; Z is the natural gas deviation factor, zero dimension; M is the gas molecular weight; r_g is the gas relative density.

The turbulent flow equation could be converted to:

$$\frac{dp}{dr} - \lambda = \frac{\mu}{k}\frac{q_{sc}}{2\pi rh}\frac{p_{sc}}{Z_{sc}T_{sc}}\frac{ZT}{p} + \beta\frac{pr_gM}{ZRT}\left(\frac{q_{sc}}{2\pi rh}\frac{p_{sc}}{Z_{sc}T_{sc}}\frac{ZT}{p}\right)^2$$

$$= \left(\frac{p_{sc}q_{sc}}{Z_{sc}T_{sc}}\right)\frac{\mu ZT}{2\pi kh}\frac{1}{rp} + \frac{Mr_g\beta ZT}{pR}\left(\frac{q_{sc}p_{sc}}{2\pi hZ_{sc}T_{sc}}\right)^2\frac{1}{r^2} \qquad (3)$$

Integrating both sides of Eq. (3), yields

$$\int_{p_{wf}}^{p_e} p\,dp = \lambda\int_{r_w}^{r_e} p\,dr + \left(\frac{p_{sc}q_{sc}}{Z_{sc}T_{sc}}\right)\frac{\mu ZT}{2\pi kh}\int_{r_w}^{r_e}\frac{dr}{r} + \frac{Mr_g\beta ZT}{R}\left(\frac{q_{sc}p_{sc}}{2\pi hZ_{sc}T_{sc}}\right)^2\int_{r_w}^{r_e}\frac{dr}{r^2} \qquad (4)$$

At present, μ, Z are usually considered as constant within the limitations, and could be put as the average value. Integrating both sides of Eq. (4), yields

$$p_e^2 - p_{wf}^2 = \lambda\int_{r_w}^{r_e} p\,dr + \left(\frac{p_{sc}q_{sc}}{Z_{sc}T_{sc}}\right)\frac{\bar{\mu}\bar{Z}T}{2\pi kh}\ln\left(\frac{r_e}{r_w}\right) + \frac{Mr_g\beta\bar{Z}T}{R}\left(\frac{q_{sc}p_{sc}}{2\pi hZ_{sc}T_{sc}}\right)^2\left(\frac{1}{r_w} - \frac{1}{r_e}\right) \qquad (5)$$

Where, $\lambda\int_{r_w}^{r_e} p\,dr$ *denotes the additional pressure drop caused by threshold pressure gradient.*

Given

$$A = \frac{p_{sc}}{Z_{sc}T_{sc}}\frac{\overline{\mu Z}T}{2\pi kh}\ln\left(\frac{r_e}{r_w}\right)$$

$$B = \frac{Mr_g\beta\bar{Z}T}{R}\left(\frac{p_{sc}}{2\pi hZ_{sc}T_{sc}}\right)^2\left(\frac{1}{r_w} - \frac{1}{r_e}\right) \qquad (6)$$

Eq. (5) is converted to:

$$p_e^2 - p_{wf}^2 - \lambda\int_{r_w}^{r_e} p\,dr = Aq_{sc} + Bq_{sc}^2 \qquad (7)$$

It is easily seen that the additional pressure drop is not a constant but is in relation with average formation pressure distribution, and with the testing production.

Because the relation equation of p versus r is hard to be obtained, the additional pressure drop can't be obtained through conventional integration method. We can only obtain the approximate solution of additional pressure drop through approximate solution of definite integral.

Fig. 2 shows that the additional pressure drop can be considered as summation of A(I) and A(II), $A = A(\text{I}) + A(\text{II})$.

Through trapezoidal approximation integral method, yields:

$$\lambda\int_{r_w}^{r_e} p\,dr \approx A(\text{I}) = \lambda\left(\frac{p_e + p_{wf}}{2}\right)(r_e - r_w) \qquad (8)$$

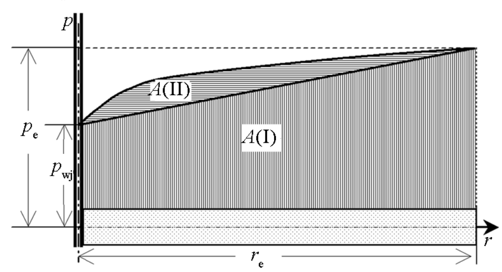

Fig. 2 The pressure profile of radial fluid flow

Because of $r_w \ll r_e$, r_w can be eliminated

$$\lambda\int_{r_w}^{r_e} p\,dr = \lambda\left(\frac{p_e + p_{wf}}{2}\right)r_e = C'(p_e + p_{wf}) \qquad (9)$$

Where, $C' = \dfrac{\lambda r_e}{2}$

Substituting Eq. (9) into Eq. (6) can obtain the deliverability equation considering threshold pressure gradient.

$$p_e^2 - p_{wf}^2 - C'(p_e + p_{wf}) = Aq_{sc} + Bq_{sc}^2 \quad (10)$$

Eq. (10) can be further converted to:

$$\left(p_e - \frac{C'}{2}\right)^2 - \left(p_{wf} + \frac{C'}{2}\right)^2 = Aq_{sc} + Bq_{sc}^2 \quad (11)$$

4 A well test data processing method

The flowing equation group can be obtained by substituting the modified isochronal test data into Eq. (11):

$$\begin{cases} \left(p_e - \frac{C'}{2}\right)^2 - \left(p_{wf1} + \frac{C'}{2}\right)^2 = Aq_{sc1} + Bq_{sc1}^2 \\ \left(p_{ws1} - \frac{C'}{2}\right)^2 - \left(p_{wf2} + \frac{C'}{2}\right)^2 = Aq_{sc2} + Bq_{sc2}^2 \\ \cdots\cdots \\ \left(p_{ws3} - \frac{C'}{2}\right)^2 - \left(p_{wf4} + \frac{C'}{2}\right)^2 = Aq_{sc4} + Bq_{sc4}^2 \end{cases} \quad (12)$$

Eq. (11) is converted in order to calculate A, B, C:

$$\left[\left(p_e - \frac{C'}{2}\right)^2 - \left(p_{wf} + \frac{C'}{2}\right)^2\right] / q_{sc} = A + Bq_{sc} \quad (13)$$

Apparently, $\left[\left(p_e - \frac{C'}{2}\right)^2 - \left(p_{wf} + \frac{C'}{2}\right)^2\right] / q_{sc}$ and q_{sc} have linear relationship, so Eq. (12) can be solved through trial-and-error procedure. Assuming various C', the curve of $\left[\left(p_e - \frac{C'}{2}\right)^2 - \left(p_{wf} + \frac{C'}{2}\right)^2\right] / q_{sc}$ versus q_{sc} can be obtained. When the relationship is linear, the corresponding C' is the solution, and the slope is B, while A can be gained through the extended flow test.

5 Practical application

The static pressure of a well in low permeability gas reservoir is 28.4 MPa, and Table 1 has the sequence of modified isochronal well test on this well.

Table 1 Sequence of modified isochronal test

Working system	p_{ws}/MPa	p_{wf}/MPa	q_g/(10^4 m^3 · d^{-1})
1	28.4	27.5	10
2	28.04	26.35	15
3	27.56	25.27	20
4	26.95	23.7	25
Extended flow test	26.22	11.65	15

With the data in Table 1, it is drawn that the deliverability curve of the previously deliverability equation which considers the additional pressure drop as a constant and the newly established deliverability equation (Fig. 3).

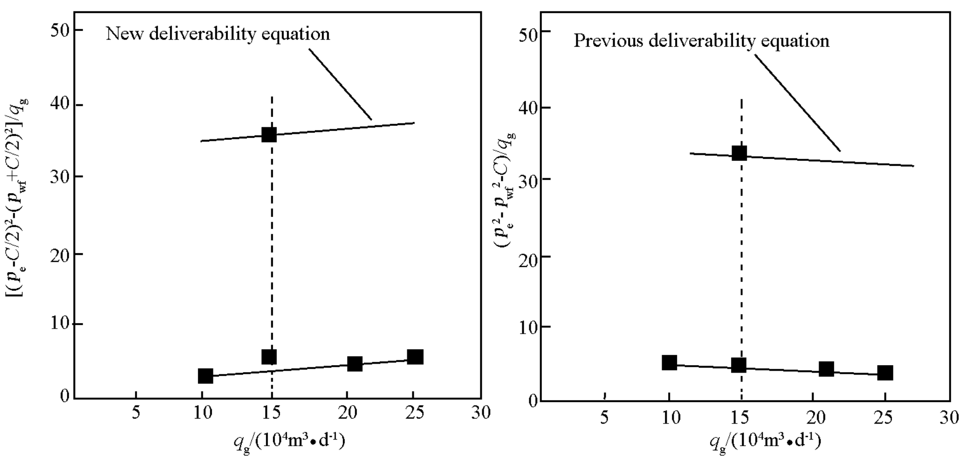

Fig. 3 Comparison between the two deliverability curves

It is obvious that the deliverability curve of previous deliverability equation have the problem of negative slope, while the result of new deliverability equation is appropriate, and the deliverability equation is:

$$(p_e - 0.151)^2 - (p_{wf} + 0.151)^2 = 33.256q_{sc} + 0.166\,1q_{sc}^2$$

The open-flow potential is q_{AOF} = 21.865 × 10^4 m^3.

6 Conclusions

1) The present deliverability equation considers the additional pressure drop caused by threshold pressure a constant, and this kind of deliverability equation could decrease the slope of deliverability curve, even to be negative.

2) Threshold pressure is introduced into the non-

Darcy flow differential equation. A new deliverability equation is obtained through approximately solving the definite integral of additional pressure drop, in which the additional pressure drop is a function of the test flow rate.

3) Through the application, the deliverability curve of present equation get negative slope problem, while the result of new equation is rational. Therefore, the new deliverability equation is more adaptable to the deliverability evaluation of low permeability gas reservoirs.

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