

The mathematical model and simulation of the electrostatic suspension system in the vertical direction

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Abstract: In order to research the vibration law of electrostatic suspension systems in the vertical direction, the mathematical model as a nonlinear differential equation is established. A series of simulation is carried out. The results show that the solution of the differential equation is a periodic function. The amplitude becomes bigger with the original velocity increased. The period becomes smaller with the original velocity increasing. The numerical methods are presented to derive the amplitude and the frequency, and the results coincide with that of the simulation. The condition during which the simple harmonic vibration arises is pointed out. The expressions for the amplitude and the period of simple harmonic vibration are derived respectively, and the results are the same with that of the simulation. This study is helpful for researching the vibration characteristics of the electrostatic suspension system. The external disturb should be controlled to lower the amplitude and the frequency of the vibration.

Key words: electrostatic suspension; mathematical model; simulation

1 Introduction

Electric suspension and magnetic suspension and are both hot spots in engineering field today. Many relative reports can be searched about them^[1]. Electric suspension is classified, according to whether or not an electric source is used in the suspension system, to be two types: active electric suspension and passive electric suspension. Passive electric suspension is also termed electrostatic suspension. Some researches were carried out on the principle of electrostatic suspension, stability, amplitude and mechanical bearing structure design^[2-5]. It was reported about the application of electric suspension in Ref. [6] and Ref. [7]. No public report is retrieved to research the nonlinear vibration characteristics of electrostatic suspension. In this paper the mathematical model of the electrostatic suspension system in the vertical direction is established, and simulation is carried out on the basis of the model to reveal the characteristics.

2 The mathematical model of the electrostatic suspension system

The principle and the force analysis of the electrostatic suspension system are shown in Fig. 1. In Fig. 1 mass points 1 and 3 are electrified with the same type and with equivalent charge. Point 1 and 3 are fixed, and it is stipulated that the positive direction is from

point 3 to 1. The coordinates of point 3 and 1 are respectively 0 and x_1 . The type of the charge of point 2 is the same as that of point 1 and 3. In the vertical direction, point 2 is in the electrostatic suspension state due to the action of the three forces; the upward electrostatic force F_{up} from point 3, the downward electrostatic force F_{down} from point 1, and the gravity mg from the earth. The coordinate of the equilibrium position is denoted by x_0 . Point 2 will vibrate due to the action of an external disturb.

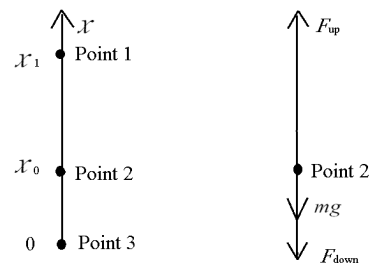


Fig. 1 The principle and the force analysis of the electrostatic suspension system

In the real system, a linear bearing is utilized to assure that point 1, 2, 3 can be on the same line. Point 1 and 3 are fixed at the two ends of the bearing, and point 2 can slide up and down the bearing. The friction between the bearing and point 2 as well as the air resistance are ignored. The motion equation of

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point 2 is

$$m \frac{d^2 x}{dt^2} = \frac{k}{x^2} - \frac{k}{(x_1 - x)^2} - mg \quad (1)$$

In Eq. (1), k is the factor due to electrostatic force (with the unit $\text{N} \cdot \text{m}^2$). k is directly proportional to the product of the charge of point 1 (or 3) and the charge of point 2. Eq. (1) is the nonlinear mathematical model of electrostatic suspension as a form of differential equation. When point 2 is located at the equilibrium position, x_0 satisfies:

$$\frac{k}{x_0^2} - \frac{k}{(x_1 - x_0)^2} - mg = 0 \quad (2)$$

Eq. (1) is a nonlinear differential equation. It is very difficult to get its general analytic solution, and numerical method is usually adopted to solve the equation^[8]. For the convenience of numerical simulation, the boundary condition of Eq. (1) is stipulated to be that, at the start time point 2 vibrates upwards with the initial velocity v_0 from the equilibrium position due to some external disturb. It is to say that, $x = x_0$, and $dx/dt = v_0 > 0$ when $t = 0$.

3 Simulation

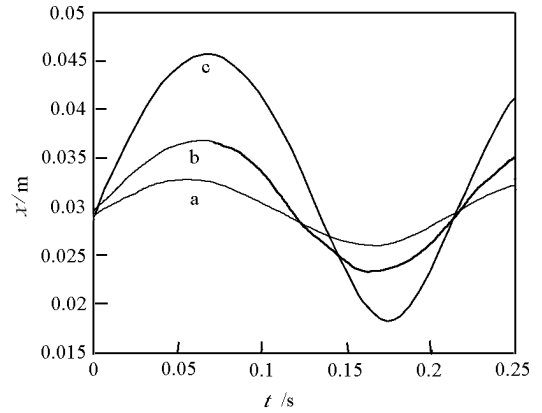
The structure parameters should be determined to discuss the motion regularity by simulation. Here let $x_1 = 0.1 \text{ m}$, $k/m = 0.01 \text{ m}^3/\text{s}^2$, and let $g = 9.8 \text{ m/s}^2$. In order to calculate x_0 , Eq. (2) is transformed as follow:

$$x_0^4 - 2x_1x_0^3 + x_1^2x_0^2 + \frac{2k}{mg}x_1x_0 - \frac{k}{mg}x_1^2 = 0 \quad (3)$$

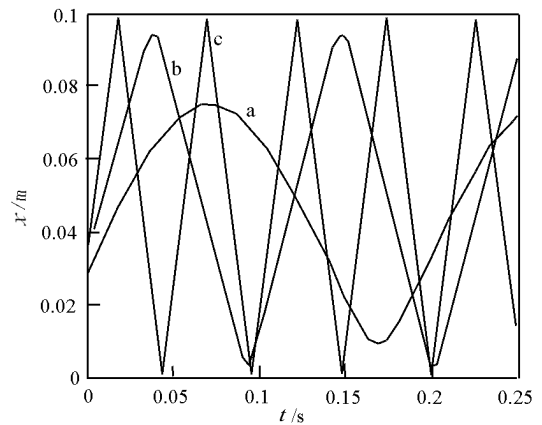
From Eq. (3), $x_0 = 0.029 \text{ m}$ can be calculated by means of the "roots" function of Matlab software^[9] (where the meaningless solutions are discarded). The "ode45" function of Matlab is selected to solve Eq. (1). The simulation results during $v_0 = (0.1, 0.2, 0.4) \text{ m/s}$ are shown in Fig. 2. From Fig. 2 it can be known that, as Eq. (1) being the motion regularity of the suspended point in the vertical direction, the solution is a periodic function. The suspended point vibrates at the two sides of the equilibrium position. But the curve form is not the same at the two sides. The amplitude becomes bigger with the increasing of the initial velocity, but the periods are 0.22 s proximately during all cases.

The simulation results during $v_0 = (1, 2, 4) \text{ m/s}$ are shown in Fig. 3. From Fig. 3 it can be known that the solution of Eq. (1) is a periodic function. The curve form is not the same during up and down toward the equilibrium position. The amplitude becomes bigger with the increasing of the initial velocity, but the amplitude is not directly proportional to the initial velocity. The periods are not identical. The larger is the

initial velocity, the smaller is the period (the higher is the frequency). But the period is not inversely proportional to the original velocity. The period (or the frequency) is related to the initial condition, and this is just the characteristic of nonlinear systems^[10].



a— $v_0 = 0.1 \text{ m/s}$; b— $v_0 = 0.2 \text{ m/s}$ c— $v_0 = 0.4 \text{ m/s}$
Fig. 2 The simulation results when
 $v_0 = (0.1, 0.2, 0.4) \text{ m/s}$



a— $v_0 = 1 \text{ m/s}$; b— $v_0 = 2 \text{ m/s}$ c— $v_0 = 4 \text{ m/s}$
Fig. 3 The simulation results when
 $v_0 = (1, 2, 4) \text{ m/s}$

4 The amplitude and the frequency characteristics of the vibration

The simulation results reveal that the suspended point vibrates periodically and asymmetrically, and the amplitude and frequency are both related to the initial condition. The numerical methods are proposed to be adopted to calculate the amplitude and the frequency as follows.

The amplitude is calculated firstly. x_u and x_d are utilized to represent the coordinates of the peak point and the valley point respectively, and the coordinate of the extreme point is generally denoted by x_{ud} . When point 2 vibrates from the equilibrium position to the

peak point or from the equilibrium position to the valley point, the following formula can be gotten from the kinetic energy theorem:

$$\int_{x_0}^{x_{ud}} \left[\frac{k}{x^2} - \frac{k}{(x_1 - x)^2} - mg \right] dx = 0 - \frac{1}{2}mv_0^2 \quad (4)$$

In Eq. (4) the function is integrable, and after integration Eq. (4) is as follows:

$$\left(-\frac{k}{x} - \frac{k}{x_1 - x} - mgx \right) \Big|_{x_0}^{x_{ud}} = -\frac{1}{2}mv_0^2 \quad (5)$$

From Eq. (5) the following formula can be derived:

$$x_{ud}^3 - \left[x_1 + \frac{k}{mgx_0} + \frac{k}{mg(x_1 - x_0)} + x_0 + \frac{v_0^2}{2g} \right] x_{ud}^2 + x_1 \left[\frac{k}{mgx_0} + \frac{k}{mg(x_1 - x_0)} + x_0 + \frac{v_0^2}{2g} \right] x_{ud} - \frac{kx_1}{mg} = 0 \quad (6)$$

The “roots” function is again used here to solve Eq. (6). Eq. (6) have three solutions, among which x_u and x_d are two meaningful solutions, and the third solution should be discarded. The solutions are shown in Table 1 when v_0 varies. For different v_0 in Table 1, x_u and x_d coincide with the simulation results shown in Fig. 2 and Fig. 3.

Table 1 The solutions of Eq. (6) when $v_0 = (0.1, 0.2, 0.4, 1, 2, 4)$ m/s

Result	v_0					
	0.1	0.2	0.4	1	2	4
x_u/m	0.0327	0.0366	0.0456	0.0751	0.0943	0.0987
x_d/m	0.0259	0.0230	0.0182	0.0094	0.0038	0.0012
The discarded solution /m	0.1205	0.1209	0.1229	0.1451	0.2846	0.8950

The frequency (or the period) is then calculated. It is necessary to derive the velocity of point 2 at any of its coordinates in order to calculate the period. v_x is here the velocity of point 2 at the coordinate x . According to the kinetic energy theorem, it can be known that

$$\int_{x_0}^x \left[\frac{k}{x^2} - \frac{k}{(x_1 - x)^2} - mg \right] dx = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_0^2 \quad (7)$$

Eq. (7) is transformed as follow after integration:

$$\left(-\frac{k}{x} - \frac{k}{x_1 - x} - mgx \right) \Big|_{x_0}^x = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_0^2 \quad (8)$$

From Eq. (8), v_x can be expressed as:

$$v_x^2 = v_0^2 + \frac{2k}{m x_0} + \frac{2k}{m (x_1 - x_0)} + 2gx_0 - \frac{2k}{m x} - \frac{2k}{m (x_1 - x)} - 2gx \quad (9)$$

The velocity of point 2 is positive in the half period during from the valley to the peak. The expression of

the period is as follow allowing to symmetry:

$$T = 2 \int_{x_d}^{x_u} \left[v_0^2 + \frac{2k}{m x_0} + \frac{2k}{m (x_1 - x_0)} + 2g x_0 - \frac{2k}{m x} - \frac{2k}{m (x_1 - x)} - 2gx \right]^{-\frac{1}{2}} dx \quad (10)$$

The “quad” function is utilized to calculate the integration in Eq. (10), and the results are shown in the Table 2, where i together with the allied numeral is the imaginary part of the number. In Table 2, the periods are all complex numbers except that during $v_0 = 4$ m/s when the period is a real number. It is explained as follow why complex numbers arise:

Table 2 The derived values of T when $v_0 = (0.1, 0.2, 0.4, 1, 2, 4)$ m/s

v_0	0.1	0.2	0.4
T/s	0.2108 - 0.0022 i	0.2053 - 0.0052 i	0.2155 - 0.0060 i
v_0	1	2	4
T/s	0.1950 - 0.0035 i	0.1093 - 0.0013 i	0.0518

The velocity is approximately 0 when point 2 is near the peak or valley, and the velocity is 0 at the peak or the valley. The time for point 2 to be near the extreme points is very short. In Eq. (10) since v_0 , x_u , x_d are approximate results of the numerical method, it is possible to extract the square root for a negative number in the integration function, and which will cause the imaginary part arises in the denominator. When the denominator becomes a real number, i will arise in the numerator. Because the time near the extreme positions is very short, the imaginary part is much smaller than the real part. So the real part can be simply regarded as the period with the imaginary part ignored.

For different v_0 in Table 1, the periods coincide with their simulation results shown in Fig. 2 and Fig. 3.

5 Simple harmonic vibration of the suspended point

When $v_0 = 0.1$ m/s, point 2 acts as simple harmonic vibration approximately, which can be known from Fig. 2. More simulation results show that when the initial velocity is very small the motion regularity of the suspended point is nearly simple harmonic vibration. In order to know the characteristics of simple harmonic vibration, the right side of Eq. (1) is expanded as a Taylor series at the point of $x = x_0$. A sinusoidal vibration equation is derived with the second order and above terms discarded:

$$m \frac{d^2 x}{dt^2} = \left[-\frac{2k}{x_0^3} - \frac{2k}{(x_1 - x_0)^3} \right] (x - x_0) \quad (11)$$

The frequency and amplitude of simple harmonic

vibration are respectively derived as follows according to Eq. (11) :

$$f_{\sin} = \frac{1}{2\pi} \sqrt{\frac{2k}{m} \left[\frac{1}{x_0^3} + \frac{1}{(x_1 - x_0)^3} \right]} \quad (12)$$

$$A_{\sin} = \frac{v_0}{\sqrt{\frac{2k}{m} \left[\frac{1}{x_0^3} + \frac{1}{(x_1 - x_0)^3} \right]}} \quad (13)$$

The frequency of simple harmonic vibration is 4.69 Hz, and the period is 0.213 s, both calculated from Eq. (12). The frequency has nothing to do with the initial condition for simple harmonic vibration. The amplitude is 0.0034 m when $v_0 = 0.1$ m/s, calculated from Eq. (13). Both the period and amplitude coincide with the simulation result shown in Fig. 2 when $v_0 = 0.1$ m/s (In Fig. 2 the amplitude is 0.0325 m - 0.0291 m = 0.0034 m).

6 Conclusions

A mass point in the electrostatic suspension state will keep static at the equilibrium position. The point will vibrate up and down around the equilibrium position due to some external disturb. A series of numerical simulation based on the differential equation shows that the curve form is not the same at the two sides of the equilibrium position. With the increasing of the initial velocity, the amplitude becomes larger, and the period becomes small. The frequency is related to the initial condition. The numerical calculation results of the amplitude and frequency coincide with that of the simulation. When the initial velocity is very small the point acts as simple harmonic vibration. The amplitude and frequency derived for simple harmonic vibration al-

so coincide with that of the simulation. The conclusions of this study are useful for systematically researching the vibration regularity of electrostatic suspension. It is also helpful for the experimental research. It is necessary in our work to control all external disturbs to a certain extent so as to lower the amplitude and frequency of the vibration.

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