

# Prediction and calculation for new energy development

Fu Yuhua<sup>1</sup>, Fu Anjie<sup>2</sup>

(1. China Offshore Oil Research Center, Beijing 100020, China;

2. Microsoft Research Asia, Beijing 100080, China)

**Abstract:** Some important questions for new energy development were discussed, such as the prediction and calculation of sea surface temperature, ocean wave, offshore platform price, typhoon track, fire status, vibration due to earthquake, energy price, stock market's trend and so on with the fractal methods (including the four ones of constant dimension fractal, variable dimension fractal, complex number dimension fractal and fractal series) and the improved rescaled range analysis (R/S analysis).

**Key words:** new energy; development; prediction; fractal method; rescaled range analysis (R/S analysis)

## 1 Introduction

The various kinds of energy development, especially the new energy development, will face many uncertain factors. If the correct prediction and calculation can be made to these factors in advance, then a great profit would be made. For example, regarding the sea water thermoelectric generation, to predict the sea surface temperature is required; in order to develop the rich new energy contained in the seabed, the offshore platform should be constructed, and it needs to carry on the prediction and calculation to the platform price; for the wind power generation, it needs to predict the gale way, in order to build more generators at the suitable place. Regarding ocean waves electricity generation, it needs precisely to calculate the movement of ocean waves. Suppose in the near future the natural disaster energy may be used, then we need to make the prediction and calculation to the fire, vibration due to earthquake and so on. Finally, for the new energy companies coming into the market, they certainly want to know the new energy price as well as the stock market's trend.

In this paper, these questions are processed with the fractal methods and the improved rescaled range analysis (R/S analysis).

## 2 Constant and variable dimension fractal prediction methods in common use and applied solid example

Recently, fractal method has been successfully used in many fields, and it is used for finding out the deeply hidden organized structure in complex phenome-

na. The quantity for reflecting the character of organized structure is called the fractal dimension, expressed with the value of  $D$ . In the fractal methods for general application at present, the fractal dimension  $D$  is a constant. For example the values of fractal dimension  $D$  for different coastlines may be taken as 1.02, 1.25 and so on. The fractal model<sup>[1]</sup> reads

$$N = \frac{C}{r^D} \quad (1)$$

where  $r$  is the characteristic scale, such as time, length, coordinates and so on;  $N$  is the object number or quantity related with the value of  $r$ , such as output, price, temperature, the value to be predicted and so on;  $C$  is a constant to be determined,  $D$  is the fractal dimension.

In the recent application of fractal method,  $D$  is the constant, may be called constant dimension fractal. It is a straight line in the double logarithmic coordinates. According to arbitrary two data points  $(N_i, r_i)$  and  $(N_j, r_j)$  on this straight line, the fractal parameters of this straight line, i. e., the fractal dimension  $D_{ij}$  and the constant  $C_{ij}$ , can be determined; Substituting the coordinates of the two data points into Eq. (1), they can be solved

$$D_{ij} = \frac{\ln(N_i/N_j)}{\ln(r_j/r_i)} \quad (2)$$

$$C_{ij} = N_i r_i^{D_{ij}} \quad (3)$$

For the straight line functional relation in the double logarithmic coordinates, it is able to process prediction and calculation with the constant dimension fractal directly.

But for the non-straight line functional relation in the double logarithmic coordinates, it is unable to

process the prediction and calculation with the constant dimension fractal. Many questions belong to this situation. In order to overcome this difficulty, the concept of variable dimension fractal in Ref. [2] ~ Ref. [4] is introduced, namely the fractal dimension  $D$  is the function of characteristic scale  $r$ .

$$D = F(r) \quad (4)$$

Now how to carry on prediction and calculation with this fractal model is to be discussed.

For the sake of convenience, let  $r$  denote the serial number of time, it will stipulate some year for the first year, then  $r_1 = 1$ , for the second year,  $r_2 = 2$ , and so on. Let  $N$  denote the given value and the value to be predicted, for example, taking  $N_1$  as the value of the first year,  $N_2$  as the value of the second year, and so on.

Now supposing that  $n$  data points are given, the values for the first year to the  $n^{\text{th}}$  year are known, thereupon the question becomes how to predict the values for the  $(n+1)^{\text{th}}$  year,  $(n+2)^{\text{th}}$  year and so on.

As a result of the  $n^{\text{th}}$  data point, namely the values of  $N_n$  and  $r_n$  for the  $n^{\text{th}}$  year are given ( $r_n = n$ ), and the value of  $r_{n+1}$  for  $(n+1)^{\text{th}}$  year is also known ( $r_{n+1} = n+1$ ), if the fractal dimension  $D_{n,n+1}$  of the constant dimension fractal decided by the  $n^{\text{th}}$  data point and  $(n+1)^{\text{th}}$  data point is known, then the value for the  $(n+1)^{\text{th}}$  year can be solved from Eq. (2)

$$N_{n+1} = N_n \left( \frac{r_n}{r_{n+1}} \right)^{D_{n,n+1}} \quad (5)$$

To this analogizes, the values for the  $(n+2)^{\text{th}}$  year and the like can be solved.

As for how to decide the fractal dimension  $D_{n,n+1}$ , it needs the information given by  $D_{12}$  (decided by the given first data point and second data point),  $D_{23}, \dots, D_{n-1,n}$  (decided by other given data points). But in general case, it is very difficult to discover the changing rule for these values of fractal dimension.

In this case, the above method cannot be used directly. The transformation of accumulated sum for the given values have to be carried on firstly, then the above method can be used to forecast the values of accumulated sum for the  $(n+1)^{\text{th}}$  year,  $(n+2)^{\text{th}}$  year and so on. Finally the values to be predicted are solved by the values of accumulated sum.

The advantage for using accumulated sum is that a sequence with increasing and decreasing can be changed into a monotone increasing sequence.

This method may be introduced as follows.

The first step, plotting the original data points  $(N_i, r_i)$  ( $i = 1 \sim n$ ) in the double logarithmic coordinates. In the ordinary circumstances they cannot fairly

agree with a constant dimension fractal model,  $N_i$  ( $i = 1, 2, \dots, n$ ) may be arranged to a fundamental sequence, namely it can be written as

$$\{N_i\} = \{N_1, N_2, N_3, \dots\} \quad (i = 1, 2, \dots, n)$$

Other sequences may be constructed according to the fundamental sequence. For example, for  $S^{(1)}$ , i. e., the sequence of first order accumulated sum,  $S_1^{(1)} = N_1$ ,  $S_2^{(1)} = N_1 + N_2$ ,  $S_3^{(1)} = N_1 + N_2 + N_3, \dots$ ; according to analogize, the sequence of second order accumulated sum, the sequence of third order accumulated sum, and the like can be constructed, namely it can be written as

$$\{S_i^{(1)}\} = \{N_1, N_1 + N_2, N_1 + N_2 + N_3, \dots\}$$

$$(i = 1, 2, \dots, n) \quad (6)$$

$$\{S_i^{(2)}\} = \{S_1^{(1)}, S_1^{(1)} + S_2^{(1)}, S_1^{(1)} + S_2^{(1)} + S_3^{(1)} \dots\}$$

$$(i = 1, 2, \dots, n) \quad (7)$$

$$\{S_i^{(3)}\} = \{S_1^{(2)}, S_1^{(2)} + S_2^{(2)}, S_1^{(2)} + S_2^{(2)} + S_3^{(2)} \dots\}$$

$$(i = 1, 2, \dots, n)$$

$$\{S_i^{(4)}\} = \{S_1^{(3)}, S_1^{(3)} + S_2^{(3)}, S_1^{(3)} + S_2^{(3)} + S_3^{(3)} \dots\}$$

$$(i = 1, 2, \dots, n)$$

It needs to point out that  $S_i^{(2)}$  denote second order accumulated sum, instead of the second power of  $S_i$ .  $S_i^{(3)}$  and the like should be comprehended similarly.

The second step, establishing the fractal models for various order accumulated sum. Taking the second order accumulated sum as an example. Plotting the data points  $(S_i^{(2)}, r_i)$  ( $i = 1 \sim n$ ) in the double logarithmic coordinates, linking these points one by one, it may result in the sectioned constant dimension fractal model. For example, according to  $n$  data points, the sectioned constant dimension fractal model composed from  $n-1$  straight lines (for different straight line, its fractal dimension is also different, this also is the simplest variable dimension fractal model), and the fractal parameters  $D_{ij}^{(2)}$ , ( $i = 1, 2, \dots, n-1, j = i+1$ ) and  $C_{ij}^{(2)}$  for each straight line can be calculated according to Eq. (2) and Eq. (3) (in which the value of  $N_i$  is replaced by  $S_i^{(2)}$ ) which means

$$D_{ij}^{(2)} = \ln(S_i^{(2)} / S_j^{(2)}) / \ln(r_j / r_i) \quad (8)$$

$$C_{ij}^{(2)} = S_i^{(2)} r_i^{D_{ij}^{(2)}} \quad (9)$$

The third step, choosing the best transformation and determining its corresponding fractal parameters. Separately drawing various order accumulated sum's data points in the double logarithmic coordinates, then choosing the best transformation (its values of fractal dimension are even increased or even decreased) and determining its corresponding fractal parameters. Because in the ordinary circumstances, the second order accumulated sum is the best, the case of second order accumulated sum will be discussed only.

After choosing the fractal model, the suitable

method should be used for deciding the fractal dimension  $D_{n,n+1}^{(2)}$  firstly, then uses the reconstructive Eq. (5) to carry on the forecast for accumulated sum. Because the values of fractal dimension are evenly increased or evenly decreased, using the following linear interpolation formula can solve the fractal dimension  $D_{n,n+1}^{(2)}$

$$D_{n,n+1}^{(2)} = 2D_{n-1,n}^{(2)} - D_{n-2,n-1}^{(2)} \quad (10)$$

For the second order accumulated sum, Eq. (5) can be expressed by

$$S_{n+1}^{(2)} = S_n^{(2)} \left( \frac{r_n}{r_{n+1}} \right)^{D_{n,n+1}^{(2)}} \quad (11)$$

For the reason that  $S_1^{(1)} \sim S_n^{(1)}$ ,  $S_1^{(2)} \sim S_n^{(2)}$  are already calculated, then the forecasting first order accumulated sum can be obtained from the forecasted second order accumulated sum, which means

$$S_{n+1}^{(1)} = S_{n+1}^{(2)} - S_n^{(2)} \quad (12)$$

Then the forecasting value can be obtained from the forecasted first order accumulated sum, which means

$$N_{n+1} = S_{n+1}^{(1)} - S_n^{(1)} \quad (13)$$

According to analogize similarly,  $N_{n+2}$ ,  $N_{n+3}$  and so on can be obtained.

It should be noted that, for some special questions, some necessary adjustments to the above-mentioned general fractal methods is needed to carry on. For example, some transformation processing should be made to the given data in advance.

Now some prediction and calculation examples are presented.

Example 1 until 08 o'clock, July 20, 1980, the tracks of No. 8007 typhoon (JOE) are given in Table 1, try to predict its future tracks<sup>[5]</sup>.

**Table 1 The given tracks of No. 8007 typhoon**

No.	Time/ m-d-h	North latitude/(°)	East longitude/(°)
1	7 16 14	10.0	147.0
2	20	11.0	146.0
3	17 02	12.0	145.0
4	08	12.7	143.8
5	14	12.3	143.2
6	20	12.5	142.0
7	18 02	13.1	140.2
8	08	13.5	138.9
9	14	14.0	137.5
10	20	14.2	136.2
11	19 02	14.3	134.7
12	08	14.7	133.1
13	14	15.0	131.7
14	20	15.2	130.1
15	20 02	15.7	128.1
16	08	16.1	126.7

With the above-mentioned fractal prediction method, the future latitudes and longitudes may be obtained respectively.

All the prediction results of this paper, the real values and the prediction results<sup>[6]</sup> are shown in Table 2 and Table 3.

**Table 2 Prediction result for the latitudes of No. 8007 typhoon**

No.	Time / m-d-h	Real value	Prediction value of this paper	Prediction value of Ref. [6]
17	7 20 14	16.3	16.4	
18	20	16.4	16.7	17.0
19	21 02	17.1	17.1	
20	08	17.4	17.4	18.2
21	14	18.1	17.8	
22	20	18.7	18.1	19.2
23	22 02	19.1	18.5	
24	08	19.5	18.8	20.0
25	14	20.1	19.2	
26	20	20.2	19.5	
27	23 02	20.4	19.9	
28	08	20.9	20.3	
29	14	20.9	20.6	
30	20	20.5	21.0	

**Table 3 Prediction result for the longitudes of No. 8007 typhoon**

No.	Time / m-d-h	Real value	Prediction value of this paper	Prediction value of Ref. [6]
17	7 20 14	125.3	125.3	
18	20	123.8	123.8	123.6
19	21 02	122.1	122.4	
20	08	120.8	121.0	120.9
21	14	119.0	119.6	
22	20	117.2	118.2	118.4
23	22 02	115.3	116.8	
24	08	113.6	115.5	116.0
25	14	112.2	114.1	
26	20	110.3	112.8	
27	23 02	108.4	111.4	
28	08	106.7	110.1	
29	14	105.3	108.9	
30	20	103.0	107.5	

From the results of Table 2 and Table 3, it is known that although the result of longitudes is not so good, generally speaking the prediction results of this paper are satisfying.

Example 2 predicting the platform price of Gulf of Mexico<sup>[7]</sup>.

Because the platform price of Gulf of Mexico cannot be predicted with the fractal method directly, therefore the following transformation to water depth  $r$

and platform price  $N$  must be carried on firstly

$$r' = ar + b$$

$$N' = AN + B$$

From this transformation, the platform price prediction formula can be obtained as in the following form:

$$N = C^* / (b - r)^{D^*} - B - B'$$

As water depth  $r = 121.92$  m, the corresponding platform price  $N = 16.7884 \times 10^6$  \$ may be obtained, to compare with the actual value  $16.5 \times 10^6$  \$, the error is only 1.7 %.

Example 3 predicting the monthly average sea surface temperature<sup>[8]</sup>.

Based on sectional variable dimension fractals, the concept of weighted fractals is presented, i. e., for the data points in an interval, their  $r$  coordinates multiply by different weighted coefficients, and making these data points locate at a straight-line in the double logarithmic coordinates. By using weighted fractals, the monthly average sea surface temperature (MASST) data on the point 30°N, 125°E of Northwest Pacific Ocean are analyzed. According to the MASST from January to August in a certain year (eight-point-method), the MASST from September to December of that year has been predicted. According to the MASST of August merely in a certain year (one-point-method), the MASST from September to December of that year has been predicted.

The MASST prediction results are as follows.

**Table 4 MASST prediction results by using eight-point-method (8PM) and one-point-method(1PM)/°C**

Year	Notes	September	October	November	December
1958	8PM	28.21	25.51	22.67	20.17
	1PM	28.24	25.55	22.72	20.22
	Real value	27.7	25.5	21.2	20
1959	8PM	28.20	25.56	22.75	20.28
	1PM	28.19	25.54	22.73	20.26
	Real value	27.6	24.7	22.9	20
1960	8PM	27.95	25.36	22.60	20.16
	1PM	28.05	25.51	22.78	20.36
	Real value	28	26	21.8	20
1961	8PM	28.70	26.14	23.37	20.91
	1PM	28.34	25.57	22.69	20.16
	Real value	28.4	26.2	22.8	22
1962	8PM	28.30	26.00	23.46	21.17
	1PM	27.90	25.48	22.83	20.47
	Real value	28	25	21	20
1963	8PM	29.36	27.86	25.78	23.80
	1PM	27.86	25.47	22.85	20.50
	Real value	27.5	24.5	21	18
1964	8PM	28.04	25.83	23.32	21.05
	1PM	27.80	25.46	22.86	20.54
	Real value	28	24.5	22	19

In addition, according to the phenomenon of frac-

tal interrelation and the fractal coefficients of this point's MASST and the monthly average air temperature of August of some points, the monthly average air temperatures of these points from September to December have also been predicted.

Example 4 predicting the stock index with variable dimension fractal.

From November 1, 2000 to February 7, 2001, the program "Daily Finance and Economics" of Beijing wired television station conducted the competition of stock index prediction. Before 13 o'clock of every business day, the participants were requested to deliver their predictions for the closing index of the same day to the television station by telephone, 2 winners of the first prize (one for Shanghai market, another for Shenzhen market), 8 winners of the second prize and 10 winners of the third prize were awarded every day. We obtained the news on November 17 and began to participate. Until the competition end on February 7, 2001, we won the first prize twice (one for Shanghai market, another for Shenzhen market), the second prize twice and the third prize seven times.

The prediction results of the stock index of Shanghai market are as follows.

**Table 5 Prediction results of the stock index of Shanghai market (1A0001)**

No.	Date	Prediction value	Real value	Error	Award
1	Nov. 16, 2000	2 087.87	2 095.98	-8.11	
2	Nov. 17, 2000	2 104.99	2 093.23	11.76	
3	Nov. 20, 2000	2 103.48	2 101.38	2.10	Third prize
4	Nov. 21, 2000	2 115.14	2 097.98	17.16	
5	Nov. 22, 2000	2 109.29	2 113.30	-4.01	
6	Nov. 23, 2000	2 125.61	2 119.43	6.18	
7	Nov. 24, 2000	2 131.43	2 053.37	78.06	
8	Nov. 27, 2000	2 048.50	2 049.67	-1.17	Third prize
9	Nov. 28, 2000	2 071.23	2 079.39	-8.16	
10	Nov. 29, 2000	2 082.63	2 067.49	15.14	
11	Nov. 30, 2000	2 063.54	2 070.61	-7.07	
12	Dec. 1, 2000	2 082.96	2 081.84	1.12	
13	Dec. 4, 2000	2 092.32	2 092.13	0.19	Second prize
14	Dec. 5, 2000	2 099.49	2 091.66	7.83	
15	Dec. 6, 2000	2 095.93	2 075.62	20.31	
16	Dec. 7, 2000	2 065.51	2 075.04	-9.53	
17	Dec. 8, 2000	2 085.09	2 073.16	11.93	
18	Dec. 11, 2000	2 044.21	2 046.07	-1.86	Third prize
19	Dec. 12, 2000	2 047.74	2 059.05	-11.31	
20	Dec. 13, 2000	2 057.62	2 056.12	1.50	
21	Dec. 14, 2000	2 055.93	2 051.07	4.86	
22	Dec. 15, 2000	2 041.31	2 039.36	1.95	
23	Dec. 18, 2000	2 026.44	2 044.54	-18.10	
24	Dec. 19, 2000	2 052.43	2 049.03	3.40	
25	Dec. 20, 2000	2 058.43	2 071.26	-12.83	
26	Dec. 21, 2000	2 084.98	2 076.89	8.09	

续表

No.	Date	Prediction value	Real value	Error	Award
27	Dec. 22, 2000	2 079.10	2 069.77	9.33	
28	Dec. 25, 2000	2 071.63	2 068.17	3.46	
29	Dec. 26, 2000	2 075.03	2 076.26	-1.23	
30	Dec. 27, 2000	2 070.66	2 058.24	12.42	
31	Dec. 28, 2000	2 057.65	2 053.70	3.95	
32	Dec. 29, 2000	2 070.41	2 073.47	-3.06	
33	Jan. 2, 2001	2 095.00	2 103.46	-8.46	
34	Jan. 3, 2001	2 121.09	2 123.89	-2.80	
35	Jan. 4, 2001	2 123.90	2 117.40	6.50	
36	Jan. 5, 2001	2 125.34	2 125.30	0.04	First prize
37	Jan. 8, 2001	2 108.06	2 102.06	6.00	
38	Jan. 9, 2001	2 098.75	2 101.13	-2.38	
39	Jan. 10, 2001	2 120.91	2 125.61	-4.70	
40	Jan. 11, 2001	2 132.74	2 119.14	13.60	
41	Jan. 12, 2001	2 106.41	2 104.74	1.67	
42	Jan. 15, 2001	2 054.82	2 032.44	22.38	
43	Jan. 16, 2001	2 003.01	2 006.88	-3.87	
44	Jan. 17, 2001	2 035.48	2 034.58	0.90	
45	Jan. 18, 2001	2 043.70	2 043.10	0.60	Third prize
46	Jan. 19, 2001	2 063.47	2 065.60	-2.13	
47	Feb. 5, 2001	2 036.62	2 008.03	28.59	
48	Feb. 6, 2001	1 960.85	1 995.31	-34.46	
49	Feb. 7, 2001	1 979.34	1 979.93	-0.59	Second prize

In the above continual 49 days' actual predictions, there were 2 days that the error less than 0.5, 5 days the error less than 1.0, 12 days the error less than 2.0, 24 days the error less than 5.0, 35 days the error less than 10.0, and 14 days the error greater than 10.0.

Obviously, this method also may be used to predict the stock price.

### 3 Two kinds of special variable dimension-fractal prediction methods

Firstly two kinds of special variable dimension fractals are introduced: fractal in fractal and high order function in fractal.

The form of fractal in fractal may be summarized as follows.

Suppose the original fractal model in Eq. (1) may be called the first order fractal, in which if the fractal dimension also is taken as the fractal form:  $D = C' / r^{D'}$ , then the fractal in fractal or the second order fractal will be formed. The higher order fractals may be deduced by analogy. In order to facilitate the idea, the different order fractals are marked with the following forms.

First order fractal:  $N_{(1)} = \frac{C_1}{r^{D_1}}$  where  $D_1 = \text{constant}$ .

second order fractal:  $N_{(2)} = \frac{C_1}{r^{D_1}}$  where  $D_1 = \frac{C_2}{r^{D_2}}$ ;

$D_2 = \text{constant}$

...

the  $n^{\text{th}}$  order fractal:  $N_{(n)} = \frac{C_1}{r^{D_1}}$  where  $D_1 = \frac{C_2}{r^{D_2}}$ ;

$D_2 = \frac{C_3}{r^{D_3}}$ ; ...  $D_{(n-1)} = \frac{C_n}{r^{D_n}}$ ;  $D_n = \text{constant}$

Similarly the high order function (function in function) may be defined. For example the original sine function (first order sine) is:  $N = \sin(kr + b)$ , if  $k = \sin(k'r + b')$ , then the sine function in sine function or the second order sine function is formed. The higher order sine function may be deduced by analogy.

In order to facilitate the idea, and consider the more widespread cases, the different order cosine function is marked with the following forms.

First order cosine function:  $\cos_{(1)} r = A_1 + B_1 \cos(C_1 + D_1 r)$  where  $D_1 = \text{constant}$

Second order cosine function:  $\cos_{(2)} r = A_1 + B_1 \cos(C_1 + D_1 r)$

Where,  $D_1 = A_2 + B_2 \cos(C_2 + D_2 r)$ ;  $D_2 = \text{constant}$

...

the  $n^{\text{th}}$  order cosine function:  $\cos_{(n)} r = A_1 + B_1 \cos(C_1 + D_1 r)$

Where,  $D_1 = A_2 + B_2 \cos(C_2 + D_2 r)$ ;  $D_2 = A_3 + B_3 \cos(C_3 + D_3 r)$ ; ...

$D_{n-1} = A_n + B_n \cos(C_n + D_n r)$ ;  $D_n = \text{constant}$

Similarly other high order trigonometric functions, high order logarithmic function, high order inverse trigonometric function, high order hyperbolic function and so on may be defined.

As the value of  $D$  in fractal model equals high order function, then the high order function in fractal is formed.

As determining the concrete forms of fractal in fractal or high order function in fractal, namely solving all the constants to be determined, the method of weighted residuals, such as the least squares method, may be used.

Suppose the number of the constants to be determined is equal to  $n$ , then we choose  $n$  given data points, their values of  $N$  are  $N_i (i = 1 \sim n)$ , after the curve fitting, the corresponding values given by the model of fractal in fractal or high order function in fractal equal  $N_i'$ , then all the constants to be determined can be solved by using the following expression:

$$S = \sum (N_i - N_i')^2 = \min$$

As determining its minimum value, the steepest descent method or other optimization method may be used.

Now the application of fractal in fractal is discussed.

As solving the practical problem, generally the fractal in fractal model with the least squares method according to the given data is established firstly, then the prediction results may be reached with the extrapolation method by using the established model.

Example 5 the world oil annual average prices from 1989 to 1993 are as follows (unit: US dollar/barrel): 17.80, 22.87, 19.33, 19.03, 16.82. Try to use the second order fractal method to establish the model of the world oil annual average prices from 1989 to 1993, and predict (extrapolate) the prices of 1994 and 1995 according to this model.

The real prices, the calculated prices from 1989 to 1993 and the predicted prices of 1994 and 1995 with the second order fractal are shown in Table 6.

**Table 6 The world oil annual average prices**

Year	$r$	Real price/\$	Calculated price/\$	Error/%
1989	1	17.80	16.73	-6.00
1990	2	22.87	22.71	-0.69
1991	3	19.33	18.94	-2.00
1992	4	19.03	17.76	-6.67
1993	5	16.82	17.29	2.79
1994	6	15.89	17.07	7.36
1995	7	17.17	16.95	-1.30

As space is limited, the application of high order function in fractal will not be discussed.

#### 4 Complex number dimension fractal prediction method

As the value of  $D$  is equal to a real number, the fractal may be called the real constant dimension fractal. In fact it is the one-dimensional model, because it only has one variable  $r$ . It is a straight line in the double logarithmic coordinates. But as the number of variable is greater than 1, then the model cannot be the real constant dimension fractal. In order to overcome this difficulty, the complex number dimension fractal and the multi-dimension fractal are proposed.

$$D, r, C, N \text{ in Eq. (1) are defined as}$$

$$D = f_1(x_1, x_2 \dots x_n); r = f_2(x_1, x_2 \dots x_n)$$

$$C = f_3(x_1, x_2 \dots x_n); N = f_4(x_1, x_2 \dots x_n)$$

Then the fractal model in  $n$ -dimensional space is obtained.

Now the complex number dimension fractal is discussed.

In Eq. (1) the values of  $D$  and  $r$  are taken as fol-

lows

$$D = a + ib$$

$$r = x + iy$$

Suppose

$$r^D = w = E + iF$$

$w$  is the multiple-valued function. Because the original fractal model is the single-valued function, it is also requested that the complex number dimension fractal is the single-valued function in this paper, namely the main value of logarithm will be taken, and it gives

$$(a + ib) \ln(x + iy) = \ln w = \ln(E + iF)$$

Therefore

$$(a + ib)(\ln|r| + i \arg r) = \ln|w| + i \arg w$$

it gives

$$\theta = \arg w = a \arg r + b \ln|r|$$

$$|w| = \exp(a \ln|r| - b \arg r)$$

Then we have

$$E = |w| \cos \theta$$

$$F = |w| \sin \theta$$

suppose

$$C = C_x + iC_y$$

$$N = N_x + iN_y$$

because

$$N = (C_x + iC_y)/(E + iF)$$

It gives

$$N_x = \frac{C_x E + C_y F}{E^2 + F^2}$$

$$N_y = \frac{C_y E - C_x F}{E^2 + F^2}$$

Thus, the related formulas for complex number dimension fractal are given. From the above inferential reasoning we can see that, in multi-dimensional space, so long as the logarithm operation is defined, the fractal model may be established.

There are 2 constants to be determined in the real fractal model:  $C$  and  $D$ . While in the complex number dimension fractal, there are 4 constants to be determined:  $C_x, C_y, a$  and  $b$ . For the real fractal model, as the number of the given data point is greater than 2, generally the least squares method should be used to determine  $C$  and  $D$ . For the complex number dimension fractal model, as the number of the given data values is greater than 4, generally the least squares method also should be used to determine  $C_x, C_y, a$  and  $b$ .

Suppose  $N_{xx}$  and  $N_{yy}$  denote the real part and imaginary part of the given data point,  $N_x$  and  $N_y$  denote the real part and imaginary part of the corresponding calculated value according to the complex number dimension fractal model, then  $C_x, C_y, a$  and  $b$  can be determined by using the following least squares method

$$\Pi = \sum [(N_x - N_{xx})^2 + (N_y - N_{yy})^2] = \min$$

In some cases, the given data points and the corresponding calculated values only have the real part (but all the other quantities  $C$ ,  $D$  and  $r$  have real part and imaginary part), thus  $C_x$ ,  $C_y$ ,  $a$  and  $b$  can be determined by using the following simplified formula

$$\Pi = \sum (N_x - N_{xx})^2 = \min$$

Example 6 a simplified method for the 5<sup>th</sup> order Stokes wave was presented<sup>[9]</sup>. In which the key quantity  $N_{xx}$  (in original text it was the non-unit parameter  $d/L$ ) is the function of parameter  $x$  (in original text it was  $K_2$ ) and  $y$  (in original text it was  $K_1$ ). The 6 given values of  $N_{xx}$  are shown in Table 7. Try to use the complex number dimension fractal model to fit these 6 data points, and to extrapolate the key quantities corresponding to other values of  $x$  and  $y$ .

For the original method to solve the key quantity, it needs to solve a complicated equation. But after fitting these 6 data points with a complex number dimension fractal model, the simple computational method may be used to extrapolate the key quantities nearby these 6 data points.

**Table 7 Key quantity  $N_{xx}$**

	$x$	0.08	0.10	0.12
$y =$	0.24	0.120 30	0.137 92	0.154 75
	0.28	0.119 22	0.136 81	0.153 54

Take the parameter for the complex form  $r = x + iy$  (for example,  $x = 0.08$ ,  $y = 0.24$  written as  $0.08 + i0.24$ ), according to the 6 given data points and the corresponding calculated values, with the steepest descent method, the optimized solution for the complex number dimension fractal model is as follows

$$D = -0.294\ 699\ 9 + i0.051\ 520\ 8$$

$$C = 0.582\ 421\ 6 + i0.842\ 528\ 5$$

Substitute the above values into the complex number dimension fractal formula, the fitting values can be calculated. The results are shown in Table 8.

**Table 8 Fitting value of  $N_x$  and the error**

$x$	$y$	$N_x$	Error/%
0.08	0.24	0.120 338 1	0.032
0.08	0.28	0.121 542 0	1.95
0.10	0.24	0.137 373 4	-0.40
0.10	0.28	0.137 033 4	0.16
0.12	0.24	0.153 971 2	-0.50
0.12	0.28	0.152 237 8	-0.85

The extrapolated results are shown in Table 9.

**Table 9 The extrapolated results**

$x$	$y$	$N_x$	$N_{xx}$	Error/%
0.08	0.20	0.120 587 2	0.121 20	-0.51
0.10	0.20	0.139 504 5	0.138 86	-0.46
0.12	0.20	0.157 734 3	0.155 77	1.26

From the fitting and extrapolated results we can see that, all the errors are less than 2 %.

## 5 Fractal series prediction method

In order to expand the application scope of fractal method, the following form of fractal series was presented<sup>[10]</sup>

$$N = \sum \frac{C_i}{r^{\rho_i}} = \frac{C_1}{r^{\rho_1}} + \frac{C_2}{r^{\rho_2}} + \dots$$

However, in this paper the special and simple form will be discussed only, namely the expanded Taylor's series in which the index is changed from the integer into the non-integer to be determined.

The fractal series as the formula of prediction (or extrapolation), may be obtained through two ways. The first one is the fitting method similar to obtain the complex number dimension fractal model; the second one is to obtain the fractal series through finding the solution of differential equation with the least squares method. In this paper the second way will be discussed only.

Example 7 for interval  $0 \leq t \leq 20$ , try to solve the following linear damped oscillation question due to earthquake, and obtain the fractal extrapolation formula.

$$X''(t) + X(t) = -2\varepsilon X'(t) \quad (\varepsilon = 0.1)$$

The initial condition reads:

$$X(0) = 0, X'(0) = 1$$

The following approximate analytical solution<sup>[11]</sup> can be found

$$X(t, \varepsilon) = \sin t - \varepsilon t \sin t + 0.5\varepsilon^2 t^2 \sin t - 0.5\varepsilon^2 t \cos t + 0.5\varepsilon^2 \sin t$$

In this paper the following fractal series of  $\varepsilon$  will be chosen (the indexes of  $\varepsilon$  are  $Y, Z, W, W$ , instead of  $1, 2, 2, 2$ ), which contains three other constants to be determined ( $A_1, A_2$  and  $A_3$ ) and already satisfies the initial condition, as the approximate analytical solution.

$$X(t) = \sin t - A_1 \varepsilon^y t \sin t + A_2 \varepsilon^z t^2 \sin t - A_3 \varepsilon^w t \cos t + A_3 \varepsilon^w \sin t$$

By using the least squares method, we have the following variational principle for solving the differential equation

$$\Pi = \int_0^{20} F^2 dt = \min$$

Where  $F = X''(t) + X(t) + 2\varepsilon X'(t)$  ( $\varepsilon = 0.1$ ) Taking the initial values as follows:  $Y = 1, Z = 2, W = 2, A_1 = 1, A_2 = A_3 = 0.5$ , with the steepest descent method, the optimized solution is as follows

$$Y = 1.048\ 389, Z = 2.223\ 077, W = 2.324\ 839,$$

$$A_1 = 0.921\ 222\ 7, A_2 = 0.327\ 744\ 9,$$

$$A_3 = 0.141\ 317\ 1$$

The results are shown in Table 10.

**Table 10 Results of  $X(t)$** 

$t$	Accurate solution	Solution of this paper	Solution of Ref. [11]
0	0	0	0
2	0.751 6	0.767 7	0.754 3
4	-0.500 9	-0.529 8	-0.505 3
6	-0.170 0	-0.165 0	-0.192 2
8	0.449 1	0.462 7	0.525 2
10	-0.185 3	-0.197 1	-0.232 8
12	-0.177 5	-0.164 6	-0.332 3
14	0.242 5	0.227 8	0.569 9
16	-0.042 7	-0.042 8	-0.120 6
18	-0.134 1	-0.122 6	-0.679 0
20	0.118 0	0.119 5	0.8767

From Table 10 we may see that, the fractal series solution agrees with the accurate solution at the whole interval, while the existing solution only agrees with the accurate solution at the interval of  $t < 10$ .

Now the fractal series will be taken as the extrapolated (predicted) formula.

From the fractal series formula the extrapolated (predicted) result for  $t = 20.5$  is as follows:  $X(20.5) = 0.136 0$ , while the accurate solution is  $0.129 3$ , the error of extrapolation formula is  $5.2\%$  only; for  $t = 21$ ,  $X(21) = 0.120 5$ , while the accurate solution is  $0.109 5$ , the error is  $10.1\%$ .

Therefore, if the required error is less than or equal to  $10.1\%$ , the above fractal extrapolation formula may be used at the interval  $[20, 21]$ .

## 6 Improved rescaled range (R/S) prediction method

The energies released by natural disaster, such as fire, earthquake and so on, are huge. If these energies can effectively use, even if a small part, the humanity also will obtain the rich energy. For the prediction to the situation of natural disaster, the improved rescaled range (R/S) analysis presented is extremely effective<sup>[12]</sup>.

H. E. Hurst is a renowned hydrology scientist. He proposed the R/S analysis or the rescaled range analysis method<sup>[13]</sup>. In which the most important work was the calculation of the Hurst exponent  $H$ .

Two new data grouping methods will be adopted to calculate the Hurst exponent  $H$ .

The first data grouping method: The number of the data in an interval is increased progressively. For example we may make the first interval to contain the data of 1950 – 1955 (there are 6 years' data altogether), the second interval contains the data of 1950 – 1956 (there are 7 years' data altogether), the rest may be deduced by analogy, the last interval contains

the data of 1950 – 2000 (there are 51 years' data altogether). It should be noted that, the Hurst exponent calculated by the data of the first interval is taken as the Hurst exponent of the year 1955, the rest may be deduced by analogy, and the Hurst exponent calculated by the data of the last interval is taken as the Hurst exponent of the year 2000.

The second data grouping method: The number of the data in an interval is fixed. For example we may make the first interval to contain the data of 1950 – 1955, the second interval contains the data of 1951 – 1956, the rest may be deduced by analogy, the last interval contains the data of 1995 – 2000, namely each interval contains 6 years' data altogether. Similar to the first method, the Hurst exponent calculated by the data of any interval is taken as the Hurst exponent of the last year of this interval.

Now the application example of the first method is presented.

Example 8 according to the fire numbers of China from 1950 to 1999, to predict the fire number of year 2000.

Firstly calculate the Hurst exponent  $H$  from 1955 to 1999. Then according to these Hurst exponents to calculate the Hurst exponent of year 2000 ( $H_{2000}$ ) with the fractal method, it gives

$$H_{2000} = 0.775 0$$

According to this calculated  $H_{2000}$ , the fire number of year 2000 may be predicted with the shooting method. Finally it gives that as the fire number of year 2000 is equal to 199 960,  $H_{2000} = 0.775 0$ . While the real value is 189 185, the predicted error is just  $5.7\%$ .

In order to effectively carry on the analysis and prediction to the statistical data<sup>[12]</sup>, the concept of high order Hurst exponent is also introduced. For all the calculated Hurst exponents, taking these values of  $H$  as the ordinary given statistical data, thereupon the R/S analysis to these data may be carried on, thus obtain a group of new Hurst exponents, and name it  $H_1$ , which is the Hurst exponent of Hurst exponent. The rest may be deduced by analogy, and may give the higher order Hurst exponents  $H_2$ ,  $H_3$  and so on. With the help of the Hurst exponent and high order Hurst exponent, whether the next year's fire number will be increased sharply can be judged.

## 7 Conclusions

This paper discusses the prediction and calculation methods established by fractal model  $N = C/r^D$  (in which the fractal dimension  $D$  may be the real number, variable and complex number), fractal series as well

as the improved rescaled range (R/S) analysis, and their applications for new energy development. In order to obtain a better effect, sometimes it needs to perform the processing to data point in advance (such as translation, accumulated sum and so on); Moreover, sometimes run the necessary adjustment to the fractal dimension  $D$ . The examples presented in this paper indicate that, these prediction and calculation methods will possibly have the good application prospect in new energy development.

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## Author

Fu Yuhua, male, born in 1945, graduated from Tangshan Railway Institute and now is a senior engineer in China offshore oil research center, Beijing, China. He has published over 100 papers. His current research is fractal theory, science of conservation of energy, hypercomplex spaces, relativity, ocean engineering, etc. He can be reached by E-mail: fuyh@cnooc.com.cn

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## Author

Cai Shenghua, male, graduated from Zhejiang University in 1996 with a bachelor degree and Kyoto University in 2005 with a mater degree. Now he is a Ph. D. student at Graduate School of Energy Science, Kyoto University, Japan. He can reached by E-mail: jpcshua@hotmail.com

Tetsuo Tezuka, male, graduated from University of Tokyo in 1983 with a doctor degree. Presently he is a professor at Graduate School of Energy Science of Kyoto University. He is a member of the Institute of Electrical Engineers of Japan, Japan Society of Simulation Technology and Japan Society of Energy and Resources.