

Study on productivity of horizontal wells with segmental perforation based on pseudo steady-state flow

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Abstract: Segmental perforation is widely used for horizontal wells. However, the flow of fluid in porous media is a complex problem. Using the Fourier transform, principle of potential superposition, trigonometric function transform, asymptotic analyses, a pressure solution of a pseudo steady-state flow model in 3D circular-boxed media has been established. Comparing with the productivity of vertical wells, an equivalence radius model can be obtained. Based on the model, a method of evaluating the productivity of segmental perforation horizontal well is presented by means of principle of superposition. It shows that the equivalence radius is different for various positions of horizontal wells; the output of both ends of horizontal wells is greater than the others under the same length of perforation interval; it is more important to obtain high productivity by increasing the length of perforation interval than enlarging the spacing between perforation intervals. The result of this research can be used to ascertain the yield of each perforated interval.

Key words: horizontal well with segmental perforation; productivity analysis; equivalent radius; dimensionless productivity index

1 Introduction

Horizontal wells are usually put into production with segmental perforation because of well drilling technology limit or special production requirement, which increases difficulty for productivity prediction of each perforated interval of horizontal wells.

Giger, et al. presented the theoretical model of horizontal wells for heterogeneous reservoirs^[1]. Joshi derived productivity equation of horizontal wells in reservoirs homogeneous in detail^[2,3]. Babu and Odeh gave productivity formula of horizontal wells in finite-box-shaped reservoirs^[4-6]. Among many researchers, Kuchuk, Ozkan and Raghavan, et al. got some constructive conclusions of single horizontal well^[7]. Subsequently, Xiong, et al. predicted productivity of the horizontal wells under various completion methods^[8]. Liu, et al. researched deliverability of horizontal wells with separated injection and production scheme^[9]. Wei, et al. made evaluation of horizontal well considering skin factor^[10]. Lei introduced a new method for productivity prediction of fractured horizontal wells^[11]. Some literatures studied the flow model of horizontal well coupling reservoir^[12,13], and others made analysis of the flowing in fractured horizontal wells^[14-16]. Liu, et al. and Zhao, et al. came up with methods for productivity calculation of horizontal wells in certain well pattern^[17].

For horizontal wells with segmental perforation, there are interactions among different perforation intervals; therefore, the position and length of perforation interval would affect productivity of horizontal wells. However, in the previous literature, horizontal wells were considered as a whole instead of perforation intervals.

In this article, firstly, the equation of productivity for horizontal wells is presented by building the mathematical model that is based on pseudo steady-state flow in 3D circular-boxed media. Secondly, by comparing with the productivity of vertical wells, an equivalence radius model of the horizontal well can be obtained; in this model perforation intervals can be considered as a series of vertical wells with corresponding equivalence radius. Thirdly, linear equations group of productivity for horizontal wells with segmental perforation is established by means of the principle of superposition, and productivity of each perforated interval is presented by solving equations, and then, the productivity of whole horizontal well is obtained.

2 3D flow model based on pseudo steady-state

The fundamental assumptions of the model are:

1) A circular-sealed heterogeneous formation with horizontal permeability and vertical permeability K_h , K_v , respectively, thickness h and drainage radius r_e .

2) A horizontal well locating in center of the formation with well length $2L$, wellbore radius r_w , vertical position z_w ; output q ;

3) Flowing are pseudo steady-state and comply with Darcy's Law.

Physical model is shown in Fig. 1.

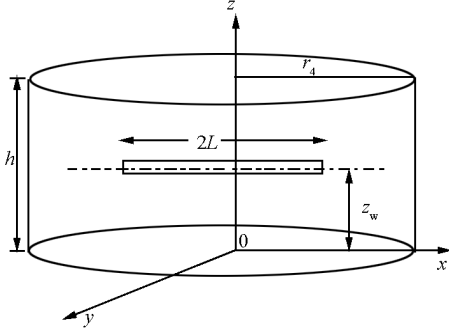


Fig. 1 The schematic map of horizontal well in circular media

Dimensionless quantities are defined as follows:

$$p_{jD} = \frac{K_h h (p_i - p_j)}{1.842 \times 10^{-3} q B \mu}, j = \text{avg, w}; \beta = \sqrt{\frac{K_h}{K_v}}$$

$$L_D = \frac{L}{h} \sqrt{\frac{K_v}{K_h}}, x_D = \frac{x}{L}, y_D = \frac{y}{L}, z_D = \frac{z}{h},$$

$$z_{wD} = \frac{z_w}{h}, r_D = \frac{r}{L}, r_{eD} = \frac{r_e}{L}, r_{wD} = \frac{r_w}{L}, \varepsilon_D = \frac{\varepsilon}{h},$$

$$r_D^2 = x_D^2 + y_D^2, z_{rD} = z_{wD} + r_{wD} L_D$$

Where, p_i is initial formation pressure, MPa; B is fluid volume factor; μ is fluid viscosity, mPa·s; z is vertical coordinate, m; r is radial coordinates, m; r_e is oil drainage radius, m.

Control equation based on pseudo steady-state flow in three-dimensional fluid flow system is expressed as:

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial p_D}{\partial r_D} \right) + L_D^2 \frac{\partial^2 p_D}{\partial z_D^2} = \frac{2}{r_D^2} \quad (1)$$

with the sealed upper boundary:

$$\frac{\partial p_D(r_D, 1)}{\partial z_D} = 0 \quad (2)$$

and the sealed lower boundary:

$$\frac{\partial p_D(r_D, 0)}{\partial z_D} = 0 \quad (3)$$

and the sealed condition of outer boundary:

$$\frac{\partial p_D(r_{eD}, z_D)}{\partial r_D} = 0 \quad (4)$$

The inner boundary condition (solution of point sink is obtained by solving line sink with the dimensionless vertical length ε_D and then supposing $\varepsilon_D \rightarrow 0$) is:

$$\lim_{\varepsilon_D \rightarrow 0} \lim_{r_D \rightarrow 0} \left[r_D \frac{\partial p_D}{\partial r_D} \right] = \begin{cases} 0, & |z_D - z_{wD}| > \frac{\varepsilon_D}{2} \\ -1/\varepsilon_D, & |z_D - z_{wD}| \leq \frac{\varepsilon_D}{2} \end{cases} \quad (5)$$

Pseudo steady-state flow equation set of point sink in this cylindrical formation is composed of Eq.(1) ~ Eq.(5). Horizontal wells can be taken as infinite conductivity line sink abstractly, and according to the principle of superposition, integration of point sink is result of line sink. So, the problem under the assumption of point sink could be solved firstly, and then, the problem of line sink could be gained by means of superposition integral.

Fourier's cosine transform pair is defined as:

$$\bar{p}_D(r_D) = \int_0^1 p_D(r_D, z_D) \cdot \cos(u_n z_D) dz_D,$$

$$p_D(r_D, z_D) = \sum_n \frac{\cos(u_n z_D)}{N(n)} \bar{p}_D(u_n) \quad (6)$$

Applying the Fourier's cosine transformation and inverse transformation to Eq.(1) ~ Eq.(5), and then making twice integration along direction of well bore, pressure of wall of horizontal well is gotten:

$$p_{wD}(x_D, 0, z_D) = \frac{1}{4} \int_{-1}^{+1} \ln \frac{r_{eD}}{\sqrt{(x_D - \alpha)^2}} d\alpha dx_D - \frac{1}{2} +$$

$$\sum_{n=1}^{\infty} \left[\frac{1}{2} \int_{-1}^{+1} K_0(\varepsilon_n \sqrt{(x_D - \alpha)^2}) d\alpha dx_D - \frac{K_0(\varepsilon_n r_{eD})}{I_0(\varepsilon_n r_{eD})} \frac{1}{2} \int_{-1}^{+1} I_0(\varepsilon_n \sqrt{(x_D - \alpha)^2}) d\alpha dx_D \right] \times$$

$$\cos(u_n z_{wD}) \cos(u_n z_D) \quad (7)$$

Application of integration and asymptotic analysis to special function yields:

$$p_{wD}(x_D, 0, z_D) \approx \ln \frac{r_{eD}}{2} + 1 -$$

$$\frac{1}{2L_D} \ln \left[2\pi r_{wD} L_D \sin \pi z_{wD} \right] -$$

$$\frac{1}{2L_D^2} \left[-z_{wD} + z_{wD}^2 + \frac{1}{3} \right] \quad (8)$$

Substituting dimensionless expressions to Eq.(8), dimension productivity equation of horizontal well q_{pss} is expressed as:

$$q_{pss} = \frac{K_h h (p_{avg} - p_w)}{1.842 \times 10^{-3} \mu B F_{pss}}, L_D \geq \frac{1.8}{\pi} \quad (9)$$

Where, p_{avg} is present average formation pressure, MPa; p_w is bottom hole pressure, MPa and formation resistance factor F_{pss} is defined as:

$$F_{pss} = \ln \left[\frac{r_e}{2L} \right] + 1 - \frac{h\beta}{2L} \ln \left[\frac{\pi r_w (1 + \beta)}{h\beta} \sin \frac{\pi z_w}{h} \right] -$$

$$\frac{h^2 \beta^2}{2L^2} \left[\frac{1}{3} - \frac{z_w}{h} + \frac{z_w^2}{h^2} \right] \quad (10)$$

Eq.(9) is simpler compared with productivity equation presented by Babu and Odeh. Moreover, it is consistent with pseudo steady-state solution obtained by Wang^[19]. In condition of Darcy flowing, Eq.(9) is compared with productivity formula of vertical well.

When the output is equal, equivalent wellbore radius is obtained.

Comparing Eq. (9) to pseudo steady-state productivity equation of vertical well, we get:

$$\ln\left[\frac{r_e}{r_w}\right] - \frac{3}{4} = \ln\left[\frac{r_e}{2L}\right] + 1 - \frac{h\beta}{2L} \ln\left[\frac{\pi r_w(1+\beta)}{h\beta} \sin\frac{\pi z_w}{h}\right] - \frac{h^2\beta^2}{2L^2} \left[\frac{1}{3} - \frac{z_w}{h} + \frac{z_w^2}{h^2}\right] \quad (11)$$

Equation of equivalent wellbore radius is:

$$r_{we} = 2L \exp\left\{-\left[1.75 - \frac{h\beta}{2L} \ln\left[\frac{\pi r_w(1+\beta)}{h\beta} \sin\frac{\pi z_w}{h}\right] - \frac{h^2\beta^2}{2L^2} \left[\frac{1}{3} - \frac{z_w}{h} + \frac{z_w^2}{h^2}\right]\right]\right\} \quad (12)$$

3 Principle of superposition for multiwell pressure drawdown

In steady-flow theory, for formation with horizontal permeability K_h , when only one vertical well produces, distribution of its pressure is:

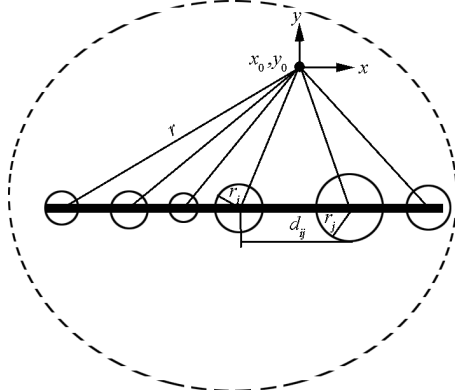
$$p(r) - C = \frac{q_i B \mu}{2\pi K_h h} \ln r \quad (13)$$

and when multi-wells produce together, the pressure distribution could be expressed as:

$$p(r) - C = \sum_i \frac{(\pm q_i) B \mu}{2\pi K_h h} \ln r_i \quad (14)$$

Where, C is undetermined constant, MPa.

For horizontal wells with segmental perforation, according to Eqs. (12), each perforation interval can be considered as a vertical well that has corresponding equivalent radius, and spacing between perforation intervals is transformed to distance between corresponding vertical wells, as shown in Fig. 2.



d_{ij} —Spacing between perforation interval i and perforation interval j , m

Fig. 2 The plan view of superposition for multiwell pressure drawdown

Assuming that all wells have the same bottom pressure and pressure loss in wellbore is ignored, there is an equation group as follows:

$$p_w - C = \frac{\mu B}{2\pi K_h h} (q_1 \ln r_{we1} + q_2 \ln d_{21} + q_3 \ln d_{31} + \dots + q_N \ln d_{N1}) \quad (15-1)$$

$$p_w - C = \frac{\mu B}{2\pi K_h h} (q_1 \ln d_{12} + q_2 \ln r_{we2} + q_3 \ln d_{32} + \dots + q_N \ln d_{N2}) \quad (15-2)$$

$$p_w - C = \frac{\mu B}{2\pi K_h h} (q_1 \ln d_{13} + q_2 \ln d_{23} + q_3 \ln r_{we3} + \dots + q_N \ln d_{N3}) \quad (15-3)$$

$$p_e - C = \frac{\mu B}{2\pi K_h h} (q_1 \ln r_e + q_2 \ln r_e + q_3 \ln r_e + \dots + q_N \ln r_e) \quad (15-4)$$

Where, r_{we} is equivalent radius, m; p_e is formation boundary pressure, MPa.

Transforming equation group (15) into equation group (16), which is written as:

$$0 = J_1 \ln \frac{r_{we1}}{d_{12}} + J_2 \ln \frac{d_{21}}{d_{we2}} + J_3 \ln \frac{d_{31}}{d_{32}} + \dots + J_N \ln \frac{d_{N1}}{d_{N2}} \quad (16-1)$$

$$0 = J_1 \ln \frac{r_{we1}}{d_{13}} + J_2 \ln \frac{d_{21}}{d_{23}} + J_3 \ln \frac{d_{31}}{r_{we3}} + \dots + J_N \ln \frac{d_{N1}}{d_{N3}} \quad (16-2)$$

$$0 = J_1 \ln \frac{r_{we1}}{d_{14}} + J_2 \ln \frac{d_{21}}{d_{24}} + J_3 \ln \frac{d_{31}}{d_{34}} + \dots + J_N \ln \frac{d_{N1}}{d_{N4}} \quad (16-3)$$

.....

$$1 = J_1 \ln \frac{r_e}{r_{we1}} + J_2 \ln \frac{r_e}{d_{21}} + J_3 \ln \frac{r_e}{d_{31}} + \dots + J_N \ln \frac{r_e}{d_{N1}} \quad (16-4)$$

Where, dimensionless productivity index J_i is defined as:

$$J_i = \frac{q_i \mu B}{542.87 K_h h (p_e - p_w)}, i = 1, 2, \dots, N \quad (16-5)$$

Where, q_i is output of well i , m^3/d .

Dimensionless productivity index of each single well could be determined by solving equation group above, and then productivity of each well can be obtained.

4 Analysis of productivity calculation of horizontal well with segmental perforation

Productivity calculation of horizontal wells with segmental perforation could be analyzed further with following examples. The assumption of the model is:

Reservoir thickness is 10 m, $K_h = 100$ mD, $K_v = 10$ mD, oil drainage radius is 500 m. A horizontal well is located in center of the reservoir, that length is 400 m.

4.1 Influence of horizontal well position on equivalent radius

According to Eq.(12), equivalent radius can represent capability for increasing production, and the production decreases along with equivalent radius of eccentric horizontal well decreasing. When horizontal well is located in the vertical center of the reservoir, the equivalent radius is the maximum.

Let $L = 100$ m, $r_w = 0.1$ m, $h = 10$ m, $K_h/K_v = 10$, if vertical position of single horizontal well changes, the equivalent radius changes as shown in Fig. 3.

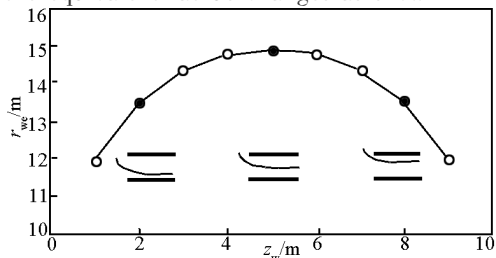


Fig. 3 Affection of horizontal well position on equivalent radius

4.2 Horizontal well with equal length and spacing of perforation interval

There are three perforation intervals with length 80 m, then the equivalent radius of each perforation interval is 14.9 m, and spacing between each other is 180 m, then $d_{12} = d_{23} = 180$ m, $d_{13} = 360$ m. Comparing with dimensionless productivity index calculated by Борисов' equation, it is as shown in Fig. 4.

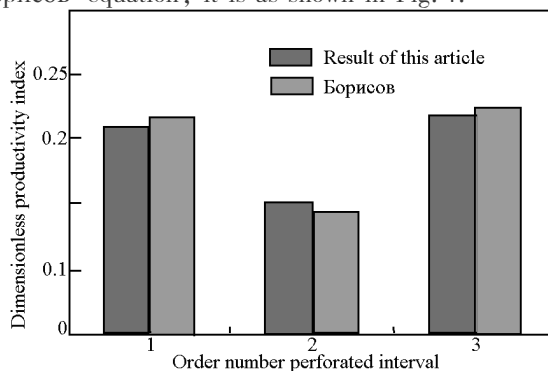


Fig. 4 Dimensionless productivity index for equal length and spacing of perforation intervals

The comparison shows that the controlled reserve and drainage area of each perforation intervals are different from various positions of perforation intervals. Dimensionless productivity index of the middle of horizontal well is the minimum, and dimensionless productivity index of the two ends are greater.

4.3 Horizontal wells with equal length and unequal spacing of perforation intervals

Let perforation interval length is 80 m, the spacing between each other is respectively: $d_{12} = 180$ m, $d_{23} = 220$ m, $d_{13} = 400$ m. The comparison is as shown in Table 1.

Table 1 Dimensionless productivity index for different spacing of perforation interval

Item	$d_{12} = d_{23} = 180$ m $d_{13} = 360$ m	$d_{12} = 180$ m, $d_{23} = 220$ m $d_{13} = 400$ m
Perforated interval 1	0.218	0.219
Perforated interval 2	0.158	0.160
Perforated interval 3	0.218	0.224

The result indicates that enlarging the spacing between perforation intervals would increase productivity index, but increasing amplitude is not large.

4.4 Horizontal wells with unequal perforation length and equal spacing

The spacing is 180 m. the length is respectively: 80 m, 100 m, 120 m, the comparison is shown as Fig. 5.

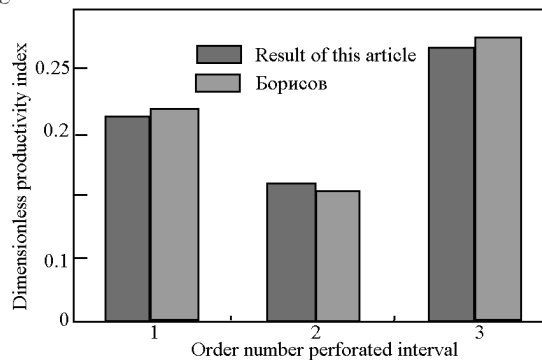


Fig. 5 Dimensionless productivity index for unequal length and equal spacing perforated interval

The Fig. 5 demonstrates, as perforation length increases, dimensionless productivity index increases gradually, especially the end sections (perforation interval 3).

The comparison between the results calculated by method of this article and by method of Борисов indicates, dimensionless productivity index obtained by the two methods are consistent, and the fractional error is just 3.9 %.

5 Conclusions

It can be concluded from the above analysis:

1) Dimensionless productivity index is related with productivity, pressure difference, reservoir thickness, permeability, oil viscosity and volume factor.

And with other parameters unchanged, dimensionless productivity index is directly proportional to productivity. If dimensionless productivity index is determined, productivity of each perforated interval will be calculated easily.

2) Vertical position of horizontal well influences equivalent radius. When horizontal well is located in vertical center of the reservoir, the equivalent radius is the largest. So if there is no influence of edge and bottom aquifer, horizontal wells should better be placed in center of reservoir.

3) When a horizontal well with segmental perforation is put into production, dimensionless productivity index of its two ends are larger than that of the middle. Therefore, it is recommended that the two ends of horizontal wells should be perforated for production.

4) The corresponding drainage area and dimensionless productivity index of each perforation interval increase along with enlarging of spacing between perforation intervals, but increasing amplitude is small. Increasing length of perforation interval can enhance productivity index obviously, so increasing perforation length is more effective for increasing productivity of horizontal well with segmental perforation.

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