

The Principle of Interaction between Plastic Volumetric and Shear Strains for Unsaturated Soils

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Abstract: The principle of interaction between plastic volumetric and shear strains for rock and soil has been extended to the field of unsaturated soils. Two new interactions of suction – plastic volumetric strain and pore air pressure – plastic volumetric strain appear in the unsaturated state of a soil except the interaction between plastic volumetric and shear strains. It is very important to find that the suction possesses a dual property, which is the origin of generating its special functions. Thereby the effect of the suction on volumetric strain includes two opposite aspects. By means of this property of suction, the physical significance of effective stress parameter, effects of suction on volume change and preconsolidation pressure, and the mechanism of collapse upon wetting all can be explained. In addition, it is theoretically proved by application of this principle of interaction that the critical state line for unsaturated soils exists, and is unique and independent of the stress history.

Key Words: the principle of interaction between plastic volumetric and shear strains; unsaturated soil; matric suction; the principle of effective stress

1 Introduction

Pores in unsaturated soil contain not only water but also air, so it is a three – phase medium including solid, liquid, and gas. The mechanical response of unsaturated soils exhibits some remarkable features induced by appearance of pore air pressure. In particular, the difference between pore air and pore water pressures caused by capillary tension, that is matric suction, possesses some of special functions, which leads to the complex behavior of unsaturated soils.

In past fifty years, the investigations of unsaturated soils have significantly advanced in either theory or measurement. However, due to the highly complex behavior of unsaturated soils some of fundamental problems in theory and measurement have not been solved yet.

At present, there are two different basic approaches in the mechanics of unsaturated soils.

First, Bishop (1959) extended the principle of effective stress proposed by Terzaghi (1936) to the field of unsaturated soils and gave an expression of effective stress^[1],

$$\sigma'_{ij} = (\sigma_{ij} - u_a \delta_{ij}) + \chi(u_a - u_w) \delta_{ij} \quad (1)$$

where σ_{ij} and σ'_{ij} are total and effective stress tensors respectively; u_a and u_w are pore air and pore water pressures respectively; χ is effective stress parameter; δ_{ij} is Kronecker's delta. $\sigma_{ij} - u_a \delta_{ij}$ is called net stress and $u_a - u_w$ denotes matric suction s .

The validity of Eq (1) was questioned by some of investigators due to the geometrical interpretation of the effective stress parameter. In recent years, Khalili et al. gave a new interpretation of χ , and the relationship between effective stress parameter and suction^[2]:

$$\chi = \begin{cases} \left[\frac{s}{s_e} \right]^{-0.55} & \text{if } s \geq s_e \\ 1 & \text{if } s \leq s_e \end{cases} \quad (2)$$

where s_e is the value of suction at the transition between saturated and unsaturated states. For wetting s_e is equal to the air expulsion value s_{ex} while during drying s_e is taken as the air entry value s_{ae} .

By means of Eqs. (1) and (2), the validity of the principle of effective stress for unsaturated soils has been demonstrated through comprehensive analyses of a number of experimental data for shear strength and volume change by Khalili et al.^[3]

In addition, it is experimentally shown that the critical state line is unique in the deviatoric stress – effective mean stress plane for both saturated and unsaturated states of a soil^[3].

In the second approach, net stress and suction are considered as two independent state variables, one described at the macroscopic scale ($\sigma_{ij} - u_a \delta_{ij}$) and the other at pore scale ($u_a - u_w$)^[4]. Also, two sets of material properties were then introduced for the two stresses, respectively. This is contrast with the usual approach in continuum mechanics, in which the state variables are averaged over the elemental volume^[3]. In fact, the two stress state variables might not be all independent^[5].

The application of the Terzaghi's principle of effective stress has successfully converted a two – phase saturated soil to a single – phase and single – stress state continuum. Following this course, it is possible by applying the Bishop's expression of effective stress and that of effective stress parameter proposed by Khalili et al. that a three – phase unsaturated soil could be converted to a single stress state continuum. Thus, the investigation of behavior of unsaturated soils could conduct within the framework of continuum mechanics and continuum thermodynamics, and many of significant results for saturated soils could be naturally and conveniently extended to the field of unsaturated soils.

The author (2006) proposed the principle of interaction between plastic volumetric and shear strains for rock and soil^[6-7], i. e. in the plastic deformations of rock and soil, there exist two relatively independent deformations; volumetric and shear strains, and the complex and nonlinear interaction between them,

which is the main origin of generating the complexity and variety of deformation for rock and soil.

Applying this principle of interaction, it is explained that the fundamental characteristics of behavior for rock and soil, such as pressure sensitivity, dilatancy and dependence of stress path, all are induced by the interaction. Especially, the dependence of stress path includes not only the effect of pressure sensitivity but also that of dilatancy, which is a their combination. Based on the principle of interaction, it is theoretically proved that the critical state line exists and is unique. In addition, the general expression of constitutive relation for geotechnical materials has been derived by means of this principle and the approach of irreversible process thermodynamics, which can sufficiently reflects the basic deformation characteristics of rock and soil, and satisfies the second law of thermodynamics^[7].

In this paper, the principle of $\epsilon_v^p - \bar{\epsilon}^p$ interaction (ϵ_v^p and $\bar{\epsilon}^p$ are plastic volumetric and generalized shear strains, respectively) has been extended to the field of unsaturated soils. It is very important to find that the suction possesses a dual property, by means of which some remarkable features of unsaturated soils could be explained.

2 The $u_w - \epsilon_v^p$ Interaction in Saturated Soils

In fact, there exists the interaction between pore water pressure and plastic volumetric strain in saturated soils except the $\epsilon_v^p - \bar{\epsilon}^p$ interaction. It is known from the principle of effective stress that the effective stress controls the volume change deformation of a soil. Therefore the variation of pore water pressure can affect the volumetric strain of the soil through changing the value of effective stress. On the other hand, the volume change can induce the variation in pore water pressure.

Based on the principle of effective stress, the variation in effective stress, or the change in effective stress path, is able to reflect the $u_w - \epsilon_v^p$ interaction.

3 Three Kinds of Interactions in Unsaturated Soils

Two new kinds of interactions, $u_a - \epsilon_v^p$ and $s - \epsilon_v^p$,

appear in unsaturated soil except the $\varepsilon_v^p - \bar{\varepsilon}^p$ interaction. Fig. 1 shows three processes of interactions in the unsaturated state of a soil.

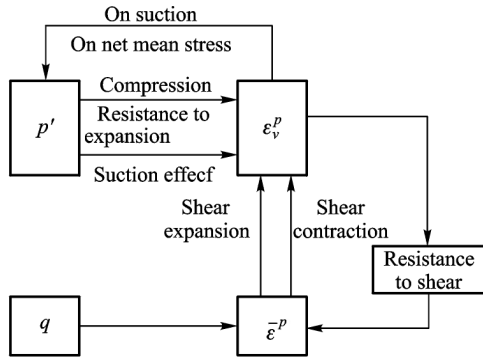


Fig. 1 Three processes of interactions in unsaturated soils

The interaction between pore air pressure and plastic volumetric strain is similar to that between pore water pressure and plastic volumetric strain in saturated soil as discussed in Section 2.

Since the capillary tension is inversely proportional to the radius of the capillary, the plastic volumetric strain directly affects the suction, whereas the suction plays dual role in its effect on ε_v^p .

Herein, the author has found that the suction possesses a dual property that is the origin of generating its special functions. The suction is able to induce the contraction of solid skeleton of a soil, so it is taken as a constituent part of effective stress. However, it is directly observed from the soil – water characteristic curve that the water content of a soil decreases monotonically with increasing suction if the suction is greater than the air entry value, i. e. the soil enters into unsaturated state. Therefore, the suction can be taken as a measure of dry (or wet) degree of unsaturated soils.

Some experimental evidences for unsaturated soils^[8] shown that the resistance to compression of a soil increases when the water content decreases. Thus, the volume contraction decreases with increasing suction during drying. Oppositely, during wetting the resistance to compression decreases, leading to the increase of volume contraction with decreasing suction. Thereby, it can be concluded that the effect of suction on the volumetric strain includes two opposite aspects: (1) the suction as a constituent part of effective stress

affects the volumetric strain, i. e. compression and resistance to expansion; (2) at the same time it reduces the above effect.

By means of the dual property of suction, some of remarkable features of unsaturated soils could be explained as follows.

(1) The physical significance of the effective stress parameter

Khalili et al. believe that the effective stress parameter is a function of suction^[2]. It is seen from Eq. (2) that the value of χ decreases with increasing suction if the value of suction larger than the air entry value.

The term χs in Eq. (1) is a constituent part of effective stress, which is a combination of tow opposite effects of the suction, in which the effective stress parameter plays the role to modify the value of suction, i. e. it modulates the effect of suction. Based on the above conclusion and Eq. (2), we would believe that the effective stress parameter describes just the second effect of the suction.

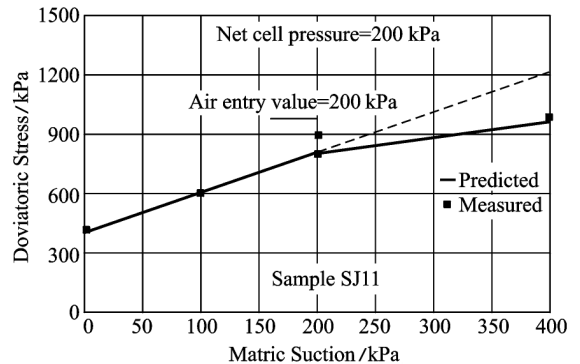


Fig. 2 Change in shear strength with suction^[3]

Fig. 2 illustrates the change in measured shear strength with matric suction^[3]. It is clearly seen from Fig. 2 that the shear strength (deviatoric stress) suddenly goes down when the value of suction is greater than the air entry value. This means that the contribution of suction to the shear strength, as a part of contribution of effective stress, decreases with increasing suction after the soil sample enters an unsaturated state. The solid line in Fig. 2 indicates the contribution of suction to shear strength.

The mechanism of generating the above experimental result could be explained in the following. It

can be known from the above analyses that the suction also is a measure of dry (or wet) degree of soil, due to which the suction possesses the second effect, i. e. the contraction of volume of a soil will decrease with increasing suction if the value of suction is greater than the air entry value. According to the principle of interaction, the plastic volumetric strain controls the change in the resistance to shear^[6]. Thus, the reduction of volumetric strain will lower the shear resistance so that the real contribution of the suction to shear strength, i. e. the contribution of χs , will decrease with increasing suction after the soil sample enter into a unsaturated state. This experimental result is a confirmation of the physical significance of effective stress parameter given above.

(2) Effects of suction on volume change

The evolution of the void ratio with suction for Jossigny loam^[3,10] is presented in Fig. 3.

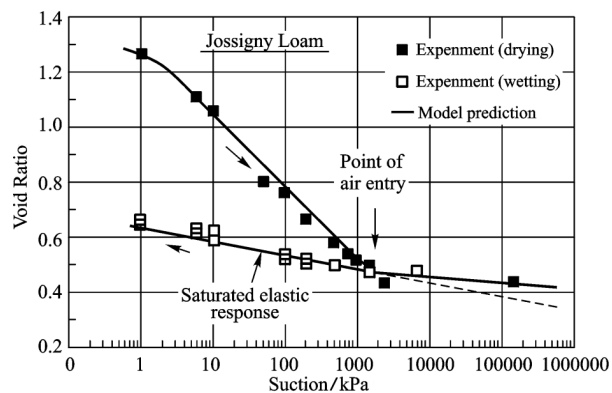


Fig. 3 Volume change with suction, Jossigny loam^[10]

The curve of void ratio – suction in Fig. 3 shows that before reaching the point of air entry, the sample undergoes a normally consolidated compression, but beyond the air entry point, a sudden reduction in the volume change with suction occurs and the sample enters into the elastic region.

This special phenomenon also originates from the dual property of suction. The increase in suction will cause two effects on the volume change of the sample: first, the suction can induce the contraction of the sample, which increases with increasing suction. At the same time, the sample is getting dry due to the increase in suction, so that the rate of contraction of the sample with suction goes down.

As discussed above, the combination of both these effects can be expressed in terms of the term χs in the expression of effective stress.

(3) Effect of suction on preconsolidation pressure

The four consolidation processes of speswhite kaolin samples at different suctions illustrated by four straight lines, C_0D_0 , C_1D_1 , C_2D_2 , C_3D_3 , respectively, are given in Fig. 4^[9], (the tests were carried out by Wheeler and Sivakumar in 1995^[11]).

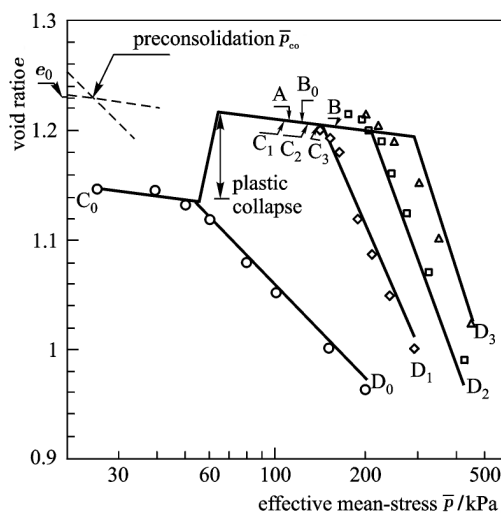


Fig. 4 Consolidation paths at different suctions for kaolin^[9]

Within each of the samples, the suction remains constant. The suctions within the four samples are equal to 0, 100, 200, 300 kPa, respectively. It can be seen from these four consolidation lines in Fig. 4 that the consolidation pressure increases with increasing suction. This is the shift in the preconsolidation pressure induced by the change in suction. According to the above analysis, the resistance to compression of the sample will increase when the suction within a sample goes up. Hence, the consolidation for each of the samples at different suctions would conduct along different consolidation paths. The larger the suction, the higher the consolidation pressure is.

(4) The mechanism of collapse upon wetting

Fig. 5 illustrates the evolution of void ratio of unsaturated speswhite kaolin sample with decreasing suction, in which a plastic collapse occurs^[9]. (the tests were performed by Wheeler and Sivakumar in 1995^[11]).

The sample is regarded as initially unsaturated since the suction within the sample is greater than the air entry

value. It can be seen from the e - s curve (e denotes void ratio) in Fig. 5 that initial path is elastic, then the void ratio suddenly drops when the value of suction attains to s^* , at which a plastic collapse takes place.

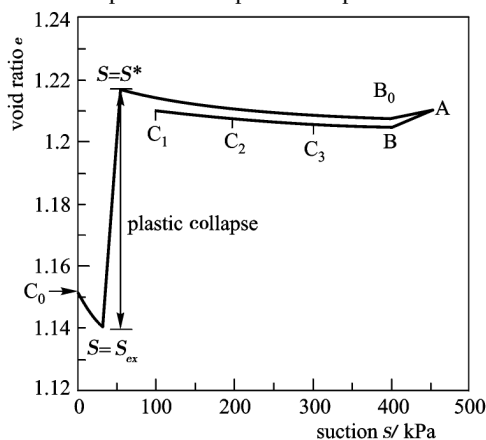


Fig. 5 Evolution of void ratio with suction^[9]

The collapse upon wetting is a process of volume contraction in collapsible soil, in which the suction continuously decreases, leading to reduction of the effective stress, which initially generates a tendency of dilation for the soil sample. Simultaneously, the sample is getting wet with decreasing suction, which leads to that the resistance to compression of collapsible sample rapidly goes down, thereby resulting in a plastic collapse.

As discussed in Section 2, the $u_w - \varepsilon_v^p$ interaction in saturated soils can be expressed in terms of the variation in effective stress path by applying the principle of effective stress. Similarly, both the $u_w - \varepsilon_v^p$ and $s - \varepsilon_v^p$ interactions in unsaturated soils could also be converted to the change in effective stress path through application of the Bishop's expression of effective stress and that of effective stress parameter given by Khalili et al.

4 Uniqueness of the Critical State line for Unsaturated Soils

Both two specimens of clay under drained and undrained triaxial compression test conditions, respectively, finally enter a special state, in which the shear strain continuously increases without further changes in the void ratio and shear resistance, termed the critical state by Hvorslev (1937)^[12].

Based on the triaxial tests for Weald clay, Roscoe et al. define the critical state line at which the Roscoe

and Hvorslev surfaces meet and at which stress paths of all triaxial compression tests end^[13].

The author has theoretically proved the uniqueness of the critical state line for saturated soils by applying the principle of $\varepsilon_v^p - \bar{\varepsilon}^p$ interaction^[6-7].

Based on the principle of interaction, the $\varepsilon_v^p - \bar{\varepsilon}^p$ interaction penetrates the whole process of soil deformation before entering into the critical state.

It can be proved that in the critical state, both elastic and plastic volumetric strains remain constant^[6-7]. This implies that a soil enters into the process of pure shear deformation.

According to the principle of interaction, the $\varepsilon_v^p - \bar{\varepsilon}^p$ interaction will end if the plastic volumetric strain holds constant, thereby leading to that the dependence of stress path, pressure sensitivity and dilatancy all disappear.

Owing to that the plastic volumetric strain controls the change in shear resistance^[6], the shear strength q_f will hold constant in the critical state, which is only dependent on the current state variables p' and e . Thus, q_f can be expressed as

$$q_f = f_1(p', e) \quad (3)$$

where p' denotes effective mean stress. Eq. (3) is independent of stress history, and is just the expression of the critical state line in the $e - p' - q$ space.

As discussed in Section 3, there are three kinds of interactions, $\varepsilon_v^p - \bar{\varepsilon}^p$, $s - \varepsilon_v^p$ and $u_w - \varepsilon_v^p$ in unsaturated soils. Notice that the plastic volumetric strain appears in all three couplings. When a unsaturated soil enters into the critical state, the three kinds of interactions all would vanish since the plastic volumetric strain has no change.

Similarly, it can be shown that the shear strength remains constant and is expressed as

$$q_f = f_2(p', e) \quad (4)$$

where p' is the effective mean stress $p' = p_{net} + \chi s$, in which p_{net} is the net mean stress $p_{net} = p - u_w$. Eq. (4) is just the expression of the critical state line in $e - p' - q$ space for unsaturated soils.

5 Conclusions

(1) The principle of interaction between plastic volumetric and shear strains has been extended to the

field of unsaturated soils. Two new kinds of interactions, suction-plastic volumetric strain and pore air pressure-plastic volumetric strain, appear in unsaturated soils except the $\varepsilon_v^p - \bar{\varepsilon}^p$ interaction.

(2) The author has found that the suction possesses a dual property, which is the origin of generating its special functions. In particular, the effect of suction on the volumetric strain includes two opposite aspects: the suction as a part of effective stress affects the volumetric strain; on the other hand, but it reduces the above effect. By means of this property, some of remarkable properties of unsaturated soils, such as the physical significance of effective stress parameter, effects of suction on volume change and preconsolidation pressure, and the mechanism of collapse upon wetting, all can be explained.

(3) Both suction-plastic volumetric strain and pore air pressure-plastic volumetric strain interactions could be converted to the change in the effective stress path by application of the Bishop's expression of effective stress.

(4) It is theoretically proved by applying the principle of $\varepsilon_v^p - \bar{\varepsilon}^p$ interaction that the critical state line exists, and is unique and independent of the stress history for unsaturated soils.

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